DETECTING SERVICE PROVIDER ALLIANCES ON THE CHOREOGRAPHY ENACTMENT PRICING GAME

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March 30, 2016

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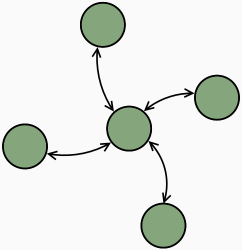
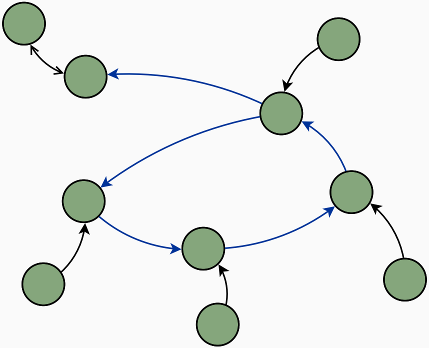
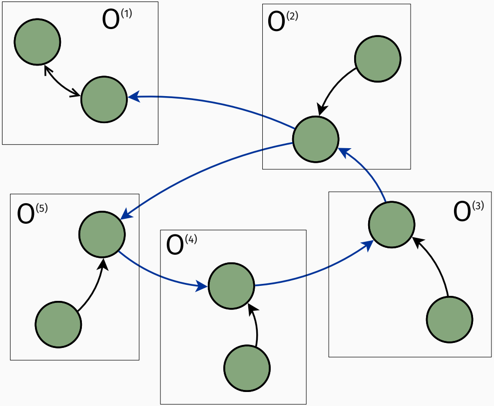
Orchestration



Choreography



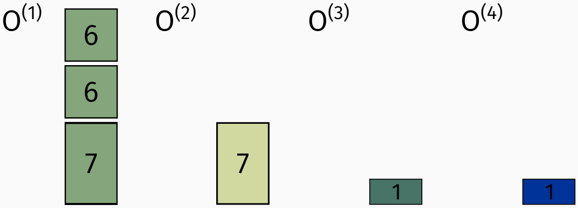
Different services may need different providers to be executed



CHARACTERISTICS

- An application is composed of several services
- The enactment of a service composition is the assignment of services to providers according to a given criteria (e.g., price)
- It is easy for an organization to delegate the execution to any provider:
    - no reason for a vendor not to subcontract resources from other vendors
    - the choreography model enforces interoperability and loose coupling
- Collaborative platform composed of resources from different organizations

- Suppose users pay a price proportional to the energy spent to execute its jobs
- Dynamic voltage and frequency scaling (DVFS)
- Energy = $\int_t P(s(t))\,\mathrm{d}t$, with $P(s(t)) = s(t)^\alpha, \alpha > 1$



$p(\{1\}) = 19^\alpha;\quad p(\{2\}) = 7^\alpha;\quad p(\{3\}) = p(\{4\}) = 1^\alpha$

cost without cooperation $= 19^\alpha + 7^\alpha + 1^\alpha + 1^\alpha$

COST OF COLLABORATION

- Suppose users pay a price proportional to the energy spent to execute its jobs
- Dynamic voltage and frequency scaling (DVFS)
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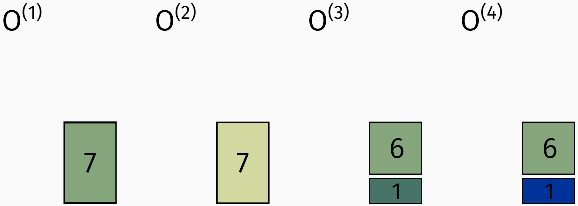
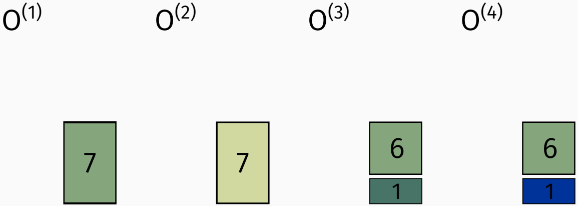


$\text{cost}^{(1)} = 7^\alpha$; $\text{cost}^{(2)} = 7^\alpha$; $\text{cost}^{(3)} = \text{cost}^{(4)} = 7^\alpha$

$\text{cost with cooperation} = 4(7^\alpha)$

Profit of coalition: $(19^\alpha + 7^\alpha + 2) - 4(7^\alpha) > 0$

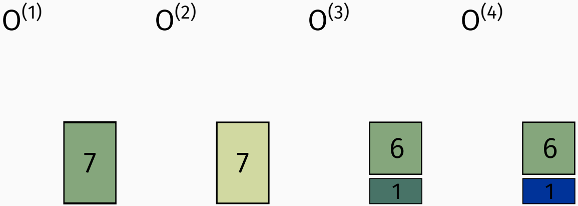


Costs for $O^{(3)}$ and $O^{(4)}$ increased from 1^α to 7^α

Members should distribute the profit and offer some compensation for them to participate.

v = cost without cooperation - cost with cooperation

$$v(\{1,2,3,4\}) = (19^\alpha + 7^\alpha + 2) - 4 \cdot 7^\alpha$$



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Let $C^{(k)}_{\mathcal{SH}}$ be the global cost of the cooperative schedule $\mathcal{SH}$ for organization $O^{(k)}$. The cooperative problem can then be stated as follows:

Find $(x_1, \ldots, x_N)$ such that $C^{(k)}_{\mathcal{SH}} - x_k \leq p(\{k\})$ and $\sum_i x_i \leq v([N])$ $\forall k (1 \leq k \leq N)$, if such vector exists.

The vector x represents the payment for each organization to have incentive to collaborate.

- The choreography enactment game models the cooperative game played by organizations
- Their main objective is to form alliances in order to schedule all jobs belonging to them at the lowest cost
- The alliance must be *stable*, i.e., no player or subset of players have incentive to leave the alliance

Cooperative game

- pair $([N], v)$ where $[N] = \{1, \dots, N\}$ is a finite set of players
- $v : 2^{|N|} \to \mathbb{R}$ is the *characteristic function*, a mapping a alliance $\mathcal{C} \subseteq [N]$ to its payment $v(\mathcal{C})$
- $v(\mathcal{C})$ is the value that $\mathcal{C}$ could obtain if they choose to cooperate

$$v(\{1,2,3,4\}) = (19^\alpha + 7^\alpha + 2) - 4 \cdot 7^\alpha \quad v(\{2,3,4\}) = (7^\alpha + 2) - 3 \cdot 3^\alpha$$
$$v(\{1,3,4\}) = (19^\alpha + 2) - 3 \cdot 7^\alpha \quad v(\{3,4\}) = 0$$
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- The problem is then to find where there is an alliance where no one can be excluded without decreasing the other player's profit
- In Game Theory, this is given by the notions of objections and counter-objections

Objection

A pair $(\mathcal{P}, y)$ is said to be *objection of i against j* if:

- $\mathcal{P}$ is a subset of $[N]$ such that $i \in \mathcal{P}$ and $j \notin \mathcal{P}$ and
- if y is a vector in $\mathbb{R}^{[N]}$ such that $y(\mathcal{P}) \leq v(\mathcal{P})$, for each $k \in \mathcal{P}$, $y_k \geq x_k$ and $y_i > x_i$ (agent i strictly benefits from y, and the other members of $\mathcal{P}$ do not do worse in y than in x).

Counter-objection

A pair $(\mathcal{Q}, z)$ is said to be a *counter-objection* to an objection $(\mathcal{P}, y)$ if:

- $\mathcal{Q}$ is a subset of $[N]$ such that $j \in \mathcal{Q}$ and $i \notin \mathcal{Q}$ and
- if z is a vector in $\mathbb{R}^{[N]}$ such that $z(\mathcal{P}) \leq v(\mathcal{P})$, for each $k \in \mathcal{Q} \setminus \mathcal{P}$, $z_k \geq x_k$ and, for each $k \in \mathcal{Q} \cap \mathcal{P}$, $z_k \geq y_k$ (the members of Q which are also members of $\mathcal{P}$ get at least the value promised in the objection).

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For organizations not changing the alliance's profit, we can show that:

Lemma 1.
Let $[N]$ *be a set of organizations and* v *be the characteristic function (corresponding to the cost savings). Let* x *be a feasible stable imputation. For each organization* $O^{(j)}$ *in* $[N]$ *such that* $v([N]) = v([N] \setminus \{j\})$, *we have* $x_j = 0$.

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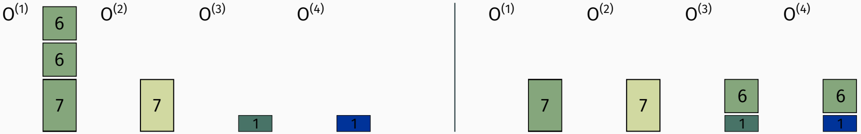

For organizations helping increase the alliance's profit

Lemma 2.

Let $[N]$ be a set of organizations and v be the characteristic function (corresponding to the cost savings). Let $O^{(i)}$ and $O^{(j)}$ be organizations such that $v([N]) > v([N] \setminus \{i\})$ and $v([N]) > v([N] \setminus \{j\})$. Let $\mathcal{O} = [N] \setminus \{j\}$ be a subset of organizations. Let $(\mathcal{O}, y)$ be an objection of $O^{(i)}$ against $O^{(j)}$. In order to have a counter-objection to $(\mathcal{Q}, z)$, with $\mathcal{Q} = [N] \setminus \{i\}$ of $O^{(j)}$ against $O^{(i)}$, a sufficient condition is:

$$x_j - x_i \leq p(\{j\}) - p(\{i\}) - cost^{(-i)} + cost^{(-j)}$$

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Intuition

Objection

$v(\mathcal{O}) = \sum_{k \in [N] \setminus \{i,j\}} y_k + y_i$

Counter-objection

$v(\mathcal{Q}) = \sum_{k \in [N] \setminus \{i,j\}} z_k + z_j$

$$v(\mathcal{Q}) - v(\mathcal{O}) \geq x_j - x_i$$

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Theorem 3.
Let $[N]$ be a set of organizations and v be the characteristic function (corresponding to the cost savings). Let A be a subset of organizations $\{j \in [N] : v([N]) > v([N] \setminus \{j\})\}$. There exists a unique stable imputation x if it fulfills all the three following conditions:

1. $\forall j \in [N] \setminus A, x_j = 0$
2. $\forall j \in A, x_j = cost^{(-j)} + p(\{j\}) - \frac{1}{|A|} \cdot \left(cost^{(A)} + \sum_{k \in A} cost^{(-k)}\right)$
3. $\forall j \in A, cost^{(-j)} + p(\{j\}) \geq \frac{1}{|A|} \cdot \left(cost^{(A)} + \sum_{k \in A} cost^{(-k)}\right)$

Corollary 4 (a lower bound for non-empty bargaining sets).
Let $[N]$ be a set of organizations and v be the characteristic function (corresponding to the cost savings). Let A be a subset of organizations $\{j \in [N] : v([N]) > v([N] \setminus \{j\})\}$. There exists a unique stable imputation x if $p(A) \geq cost^{(A)}$.

Corollary 5.
Let $[N]$ be a set of organizations with their sets of jobs. If all organizations have as objective function $(\sum_J C_J)$ or $(\sum_J E_J)$, then Algorithm 1 determines in polynomial time whether $[N]$ can form an alliance and, if possible, returns the imputation vector.

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ALLIANCE DETECTION ALGORITHM

Input: $[N]$ of organizations, function v (cost savings), and $\text{cost}^{(\cdot)}$.

Output: (Whether there is a alliance or not and the imputation vector)

Compute the lowest cost schedule $\mathcal{SH}$

forall *organizations* $O^{(k)} \in [N]$ **do**

  Compute the lowest cost local schedule $\left(\text{cost}^{(k)}_{\text{local}} = p(\{k\})\right)$

  Compute the lowest cost schedule using all resources except $O^{(k)}$'s $\left(\text{cost}^{(-k)}\right)$

Compute the lowest cost schedule using all the resources and its cost $(\text{cost}^{([N])})$

forall *organization* $O^{(k)} \in [N]$ **do**

  Compute $p([N] \setminus \{k\})$ $\left(= \sum_{j \in [N], j \neq k} p(\{j\})\right)$

  Compute $v([N] \setminus \{k\})$ $\left(= p([N] \setminus \{k\}) - \text{cost}^{(-k)}\right)$

Compute $A = \{j \in [N] \mid v([N]) > v([N] \setminus \{j\})\}$, $p(A)$, $\text{cost}^{(A)}$ and $\sum_{k \in A} \text{cost}^{(-k)}$

if $p(A) < cost^{(A)}$ **then**

  return *(alliance=false, imputation=$\emptyset$)*

forall *organization* $O^{(k)} \in A$ **do**

  compute x_k according to Property (2) of the Theorem;

return *(alliance=true, imputation=x)*

FINAL REMARKS

- We found the basic conditions needed for (stable) alliances for problems that can be optimally solved in polynomial time
- Can we adapt this technique for other objectives like makespan? A recent work by Azar et al. (ACM Economics and Computation 2015) may be the solution
- The bargaining set of this game suggests a relation of this problem with truthful mechanisms from algorithmic mechanism design