On Bi-level approach for Scheduling problems

New challenges in Scheduling Theory – Aussois 2016

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Contents

Context

- ICT technology is evolving very fast → **Smart ICT**
- Decisions are generally not controlled by a **single decision maker**
- e.g. Cloud Computing, Internet of Things involve several decision makers
- Bi-level optimization involves **2** decision makers
- Equivalent to Hierarchical games (e.g. Stackelberg games) → **iterative games**

Bi-level problems

- Two **nested** optimization levels
- First level is defined as *"The upper level"* or *"The leader problem"*
- Second level is defined as *"The lower level"* or *"The follower problem"*
- Decision variables partitioned into **two** sets
- Decision maker can only act on their decision variables
- **However** they can act **indirectly** on each other
- The follower problem is **parametrized** by the leader decision
- The **feasibility** of the leader problem depends on the **optimality** of the follower problem

Bi-level formulation

$$
\begin{aligned}
\text{"} \min_{x, y' \in Y(x)} \text{"} \quad & F(x, y') \\
\text{s.t.} \quad & G(x, y') \leq 0 \\
& Y(x) = \operatorname*{argmin}_{y \in Y} \quad f(x, y) \\
& \text{s.t.} \quad g(x, y) \leq 0 \\
& x, y \geq 0
\end{aligned}
$$

where

- F: leader objective $\mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}$
- f: follower objective $\mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}$
- G: leader's constraints $G : \mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^p$
- g: follower's constraints $\mathbf{R}^n \times \mathbf{R}^m \to \mathbf{R}^q$

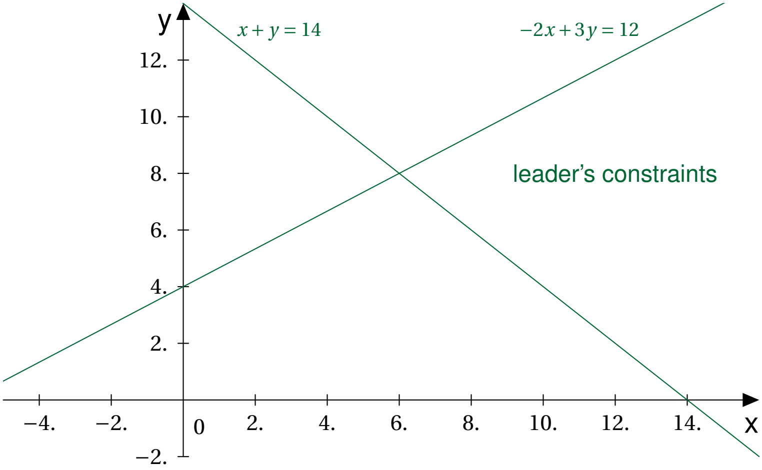
Rational reaction set

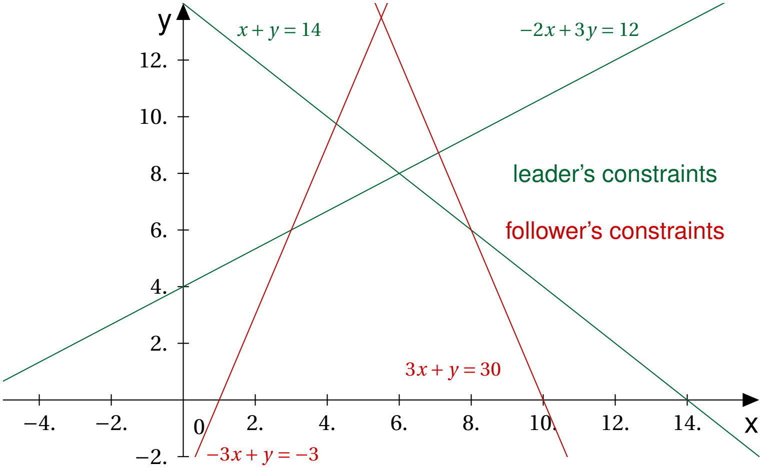
- The follower 's feasible set parametrized by $x \in X$: $S(x) = \{y \in Y : g(x,y) \leq 0\}$.
- The Follower's rational decision set:
  $Y(x) = \{y' \in Y : y' \in \text{argmin}[f(x,y) : y \in S(x)]\}$
- The Inducible Region: $IR = \{(x,y') \in S, y' \in Y(x)\}$
- For a given x, the follower reacts **optimally** *(if possible)* to leader decisions → $Y(x)$
- For a given x, the follower may have **several** optimal reactions(solutions) → $|Y(x)| > 1$
- For a given x, the follower is **indifferent** to each $y' \in Y(x)$. This is **not** the case for the Leader
- If$|Y(x)| > 1$ then two cases:
  - **Optimistic** model: " $\min\limits_{x,y' \in Y(x)}$ " → $\min\limits_{x,y' \in Y(x)}$
  - **Pessimistic** model: " $\min\limits_{x,y' \in Y(x)}$ " → $\min\limits_{x} \max\limits_{y' \in Y(x)}$

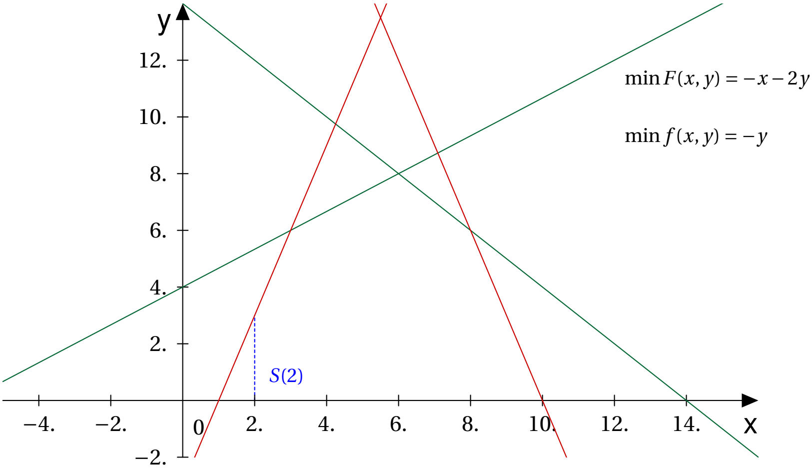
Example

Dempe et al. 2005: optimistic case

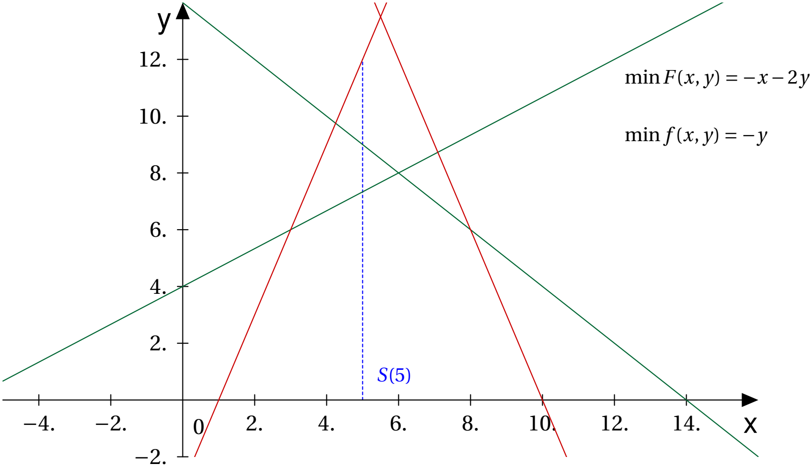
$$
\begin{aligned}
P = \min_{x\geq 0, y' \in Y(x)} \quad & F(x,y) = -x - 2y' \\
\text{s.t. } & 2x - 3y' \geq -12 \\
& x + y' \leq 14 \\
& Y(x) = \operatorname*{argmin}_{y \geq 0} \quad f(y) = -y \\
& \text{s.t.} -3x + y \leq -3 \\
& \qquad 3x + y \leq 30
\end{aligned}
$$


y
x
x + y = 14
−2x + 3y = 12
leader's constraints
12.
10.
8.
6.
4.
2.
−2.
−4.
−2.
0
2.
4.
6.
8.
10.
12.
14.


y
12.
10.
8.
6.
4.
2.
−2.
x
−4.
−2.
0
2.
4.
6.
8.
10.
12.
14.
$x + y = 14$
$-2x + 3y = 12$
$3x + y = 30$
$-3x + y = -3$
leader's constraints
follower's constraints

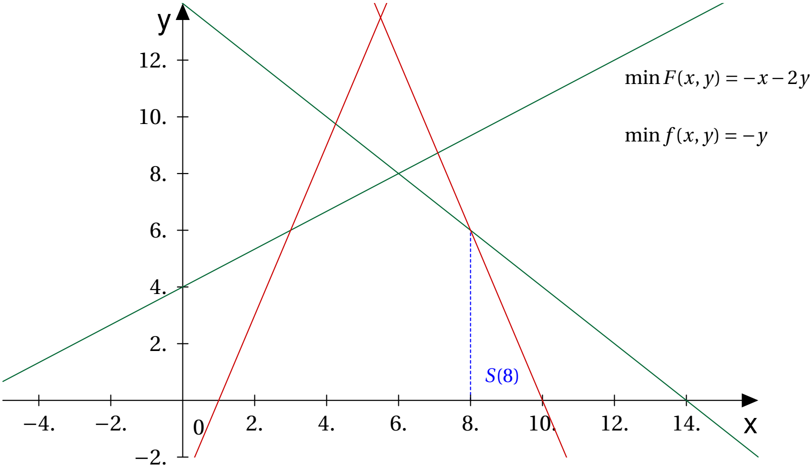


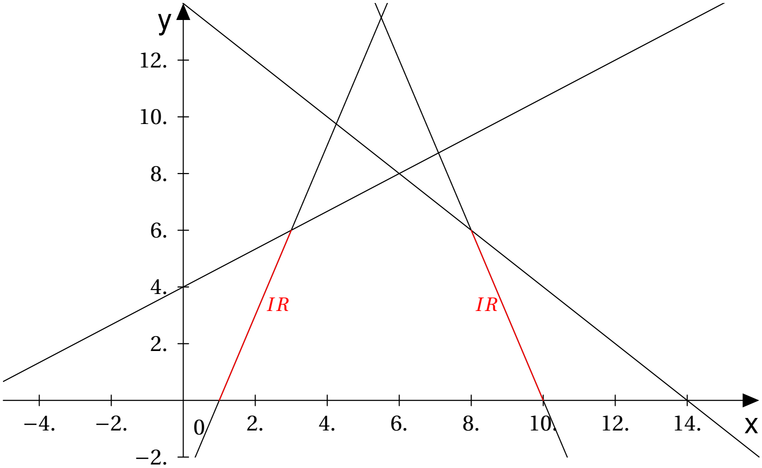
The blue dashed line is the parametrized follower decision set for $x = 2$



⚠ The follower is indifferent to leader's constraints

⚠ $Y(5)$ is not feasible for the leader


y
12.
10.
8.
6.
4.
2.
−2.
0
−4.
−2.
2.
4.
6.
8.
10.
12.
14.
x
S(8)
min F(x, y) = −x − 2y
min f(x, y) = −y


y
12.
10.
8.
6.
4.
2.
−2.
IR
IR
−4.
−2.
0
2.
4.
6.
8.
10.
12.
14.
x

State of the Art I

Dempe et al. 2005

Even for convex bi-level problems:

- The decision set may be not convex
- The decision set may be discontinued

Jeroslow et al. 1985

- The linear bi-level problem has been shown NP-hard

Resolution in continuous case

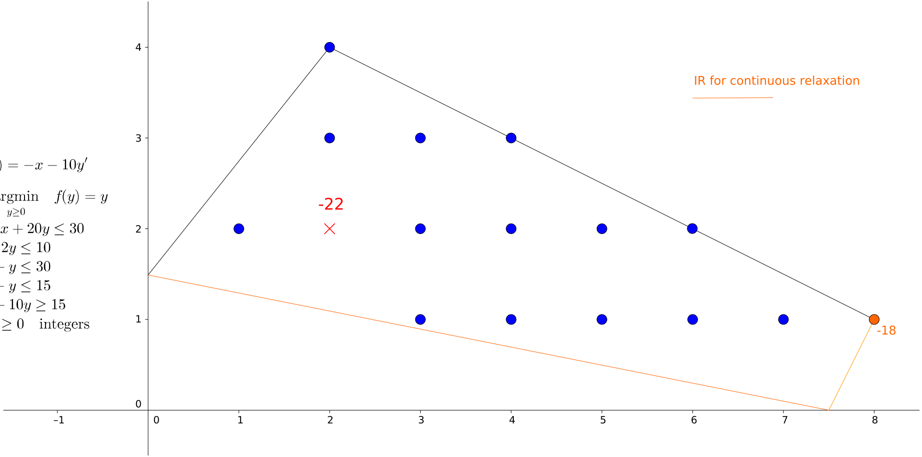
- The follower problem is mostly replaced by its Karush-Kuhn-Tucker conditions
- The resulting single-level problem is solved using non-linear optimization algorithms

State of the Art II

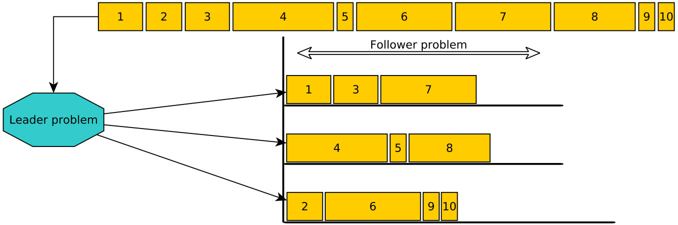
Resolution in discrete case

- No transformations
- Solution of relaxed bi-level problems does not provide valid bounds
- An integer solution (found using a Branch & Bound) is not necessary bi-level feasible
- Integer solutions does not generate sterile nodes

$$P = \min_{x\geq 0, y'\geq 0} \quad F(x,y) = -x - 10y'$$
$$\text{s.t.} \quad Y(x) = \operatorname*{argmin}_{y\geq 0} \quad f(y) = y$$
$$\text{s.t.} -25x + 20y \leq 30$$
$$x + 2y \leq 10$$
$$3x + y \leq 30$$
$$2x - y \leq 15$$
$$2x + 10y \geq 15$$
$$x, y \geq 0 \quad \text{integers}$$

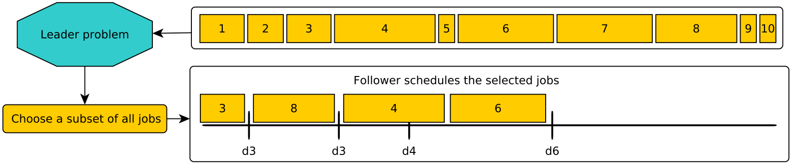


Bi-level total weighted completion time [Kis and Kovács, 2010]



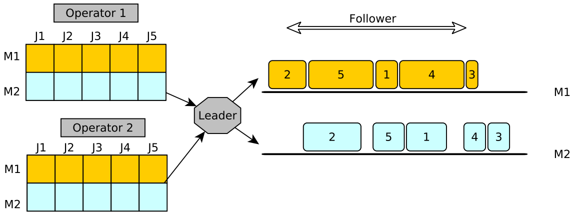
- M identical machines, N jobs
- Each jobs has a processing time p_j and two non-negative weights, w_j^1 and w_j^2
- Leader assigns jobs to machines and mimimize $\sum_j w_j^1 C_j$
- Follower schedules the assigned jobs on each machine and mimimize $\sum_j w_j^2 C_j$
- Decision problem is NP-complete in the strong sense ([Kis and Kovács, 2010])

Bi-level order acceptance [Kis and Kovács, 2010]



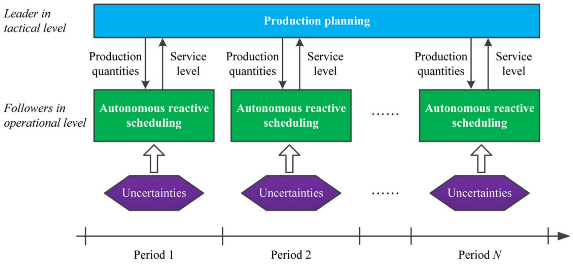
- 1 machine, N jobs
- Each jobs has a processing time p_j, a deadline d_j and two non-negative weights, w_j^1 and w_j^2
- Leader assigns jobs to machines and mimimize $\sum_j w_j^1 R_j$ with $R_j = 1$ if $C_j > d_j$
- Follower schedules the assigned jobs on each machine and mimimize $\sum_j w_j^2 C_j$
- Decision problem is NP-complete in the strong sense

Bi-level flow-shop problem [Karlof and Wang, 1996]



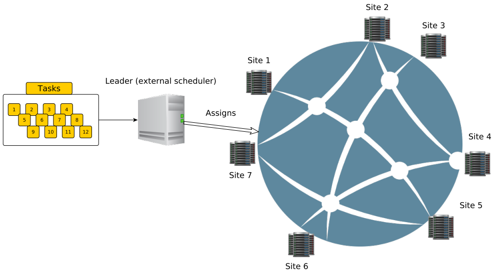
- M machine, N jobs, M operators
- Each operator has its own time table for each jobs on each machine
- Leader assigns operators to machines to mimimize the total flow-time
- Follower schedules then all jobs according to the processing time provided by the assigned operators to minimize the makespan

Scheduling under production uncertainties [Chu et al., 2015]



- Planning and scheduling are two core decision intricately linked.
- Leader −− > planning problem based on customers orders
- Followers −− > scheduling problems to provide a production schedule
- Multiple products can be manufactured in a single period, the production need to be scheduled
- The scheduling problem is parametrized by the production quantity obtained by the leader

Bi-level Grid Scheduling [Bianco et al., 2015]



- Each task has a release date and a due-date
- Leader $-->$ External scheduler assigns tasks to grid computing sites
- Followers $-->$ Multiple followers with own objectives
- Leader wants to minimize a cost proportional to the tardiness
- Followers wants to maximise the computational resource usage efficiency

Thank you

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