Structural Properties of an Open Problem in Preemptive Scheduling

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Motivation

Th. (Coffman & Graham, '72)

$P2|prec, p_j = 1|\sum C_j$ is ideal and can be solved in $O(n^2)$ time.

Th. (Coffman, Sethuraman & Timkovsky, '03)

$P2|pmtn, prec, p_j = 1|\sum C_j$ is ideal and can be solved in $O(n^2)$ time.

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$P2|prec, r_j, p_j = 1|C_{\max}$ can be solved in $O(n^{\log_2 7})$ time.

Th. (Baptiste & Timkovsky, '04)

$P2|prec, r_j, p_j = 1|\sum C_j$ can be solved in $O(n^9)$ time.

Th. (Coffman & Dereniowski & Kubiak, '12)

$P2|prec, r_j, p_j = 1|\sum C_j$ is ideal and can be solved in $O(n^3)$ time.

Problem (Considered in this presentation)

Is $P2|pmtn, prec, r_j, p_j = 1|\sum C_j$ ideal?

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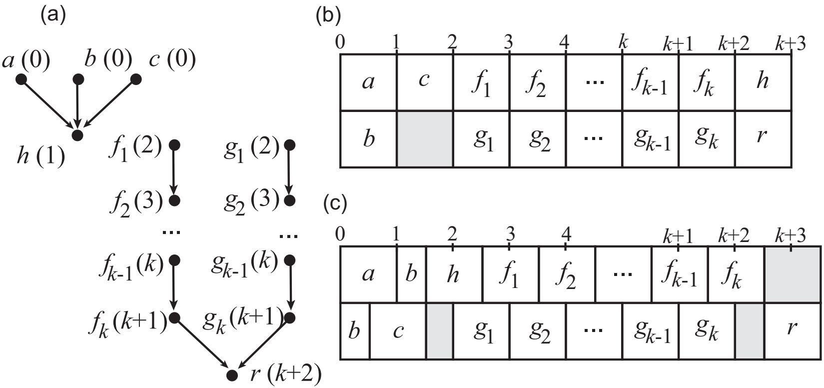
Problem (Considered in this presentation)

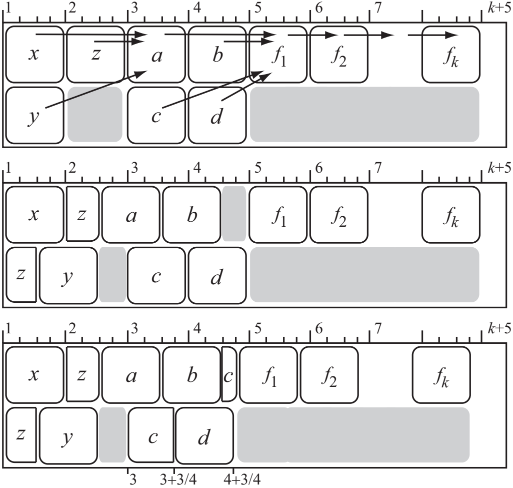
Is $P2|pmtn, prec, r_j, p_j = 1|\sum C_j$ ideal? No. Complexity - Open.

$P2|pmtn, intree, r_j, p_j = 1|\sum C_j$ is not ideal



Multiple of 1/2, 1/4, ... preemptions (?) - the main question

(a)
(b)
(c)



Events and chunks

For a given schedule $\mathcal{P}$, define a vector $\boldsymbol{e} = (e_1, \ldots, e_q)$, where $0 = e_1 < e_2 < \cdots < e_q$, such that each e_i, $i > 1$, is either:

- job start,
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The elements of $\boldsymbol{e}$ are called the *events of* $\mathcal{P}$.
The parts of $\mathcal{P}$ executing between consecutive events are called *chunks of* $\mathcal{P}$.

How do the chunks of an optimal schedule look like?

Lemma

Let $\mathcal{P}$ be an optimal schedule. If the i-th chunk is not empty, then:

1. *there exists $a \in \mathcal{J}$ such that its execution time in the chunk equals the length of the chunk, and*
2. *there are at most three jobs 'present' in the chunk, and*
3. *if three jobs are present, then one of them completes at the end of the chunk.*

Normal schedules and abnormality points

A preemptive schedule $\mathcal{P}$ with q events is *normal* if:

- i-th chunk has length that is multiple of $1/2^i$,
- the total execution time of each job in the chunk is a multiple of $1/2^{i+1}$.

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If a schedule $\mathcal{P}$ with is not normal, then the minimum index $i \in \{1, \ldots, q-1\}$ such that the i-th chunk of $\mathcal{P}$ does not satisfy the above conditions is called the *abnormality point of* $\mathcal{P}$.

How can the abnormality point (really) arise?

Given a schedule $\mathcal{P}$, for each $i \in \{1, \ldots, q-1\}$ define

$$A_i(\mathcal{P}) := \text{the set of jobs whose execution time in chunk } i \text{ is not multiple of } 1/2^{i+1}.$$

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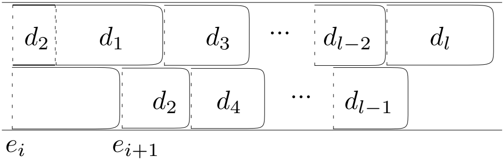

Main results

The above sequence $(d_1, \ldots, d_l)$ of jobs is called an *alternating chain*.

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The above sequence $(d_1, \ldots, d_l)$ of jobs is called an *alternating chain*.

1. we first prove that some alternating chain is present (i.e., one with two jobs),
2. we take an optimal schedule that satisfies certain (technical) properties and has maximal abnormality point,
3. we are able to prove that we can obtain a new schedule that also satisfies those properties, has the same abnormality point but its alternating chain has one more job.

Main results

Theorem

There exists a normal optimal schedule for $P2|pmtn, \textit{in-tree}, r_j, p_j| \sum C_i$.

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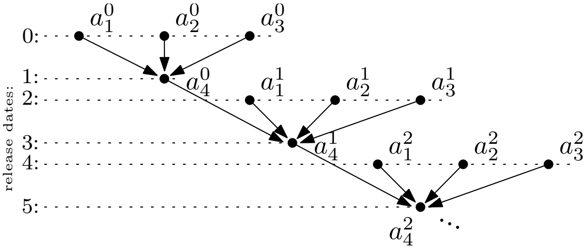
Corollary

For the given set of n jobs, there exists an optimal schedule for $P2|pmtn, in\text{-}tree, r_j, p_j|\sum C_i$ *such that each job start, preemption, resumption or completion occurs at a time point that is a multiple of* $1/2^{2n}$.

Lower bound



release dates:
0:
1:
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4:
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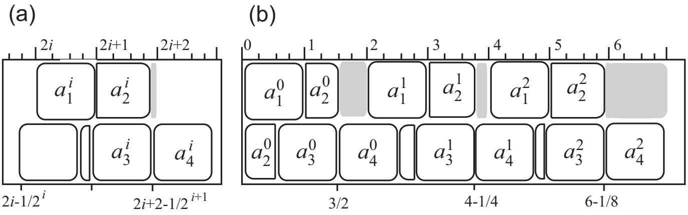
a_1^0 a_2^0 a_3^0
a_4^0 a_1^1 a_2^1 a_3^1
a_4^1 a_1^2 a_2^2 a_3^2
a_4^2
$\ddots$

Lower bound on the power (granularity)



Lower bound on the power (granularity)

Theorem

There exists a set of n jobs $\mathcal{J}$ such that there exists no optimal solution to $P2|pmtn, in\text{-}tree, r_j, p_j = 1|\sum C_i$ for $\mathcal{J}$ in which each job start, completion and preemption occurs at a time point that is a multiple of $1/2^{n/4-1}$. □

Conclusions and open questions

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Thank you