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Comparing global link arrangements for Dragonfly networks

Hastings, Rincon-Cruz, Spehlmann,
Meyers, Xu, and Bunde (Knox College)
and **Vitus Leung (Discrete Math & Opt)**

New Challenges in Scheduling Theory 2016



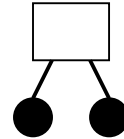
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Dragonfly

- Hierarchical architecture to exploit high-radix switches and optical links

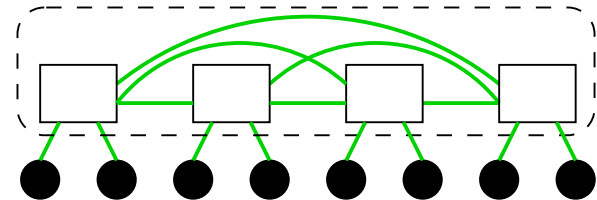
Dragonfly

- Hierarchical architecture to exploit high-radix switches and optical links
 - Nodes attached to switches



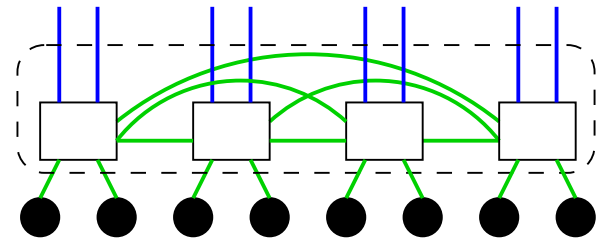
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- Hierarchical architecture to exploit high-radix switches and optical links
 - Nodes attached to switches
 - Switches form groups
 - Group members connected w/ **local edge** (electrical)



Dragonfly

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 - Each pair of groups connected w/ **global edge** (optical)



Dragonfly

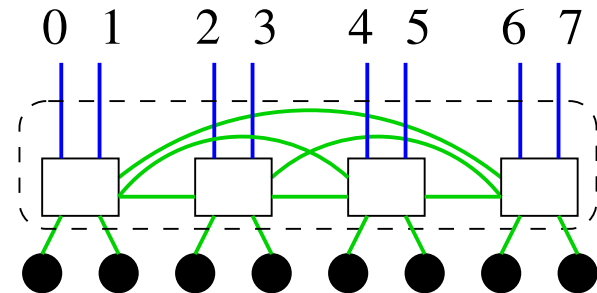
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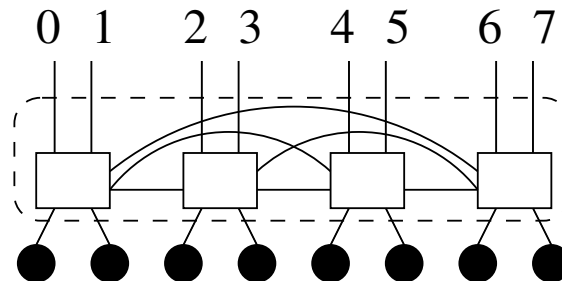
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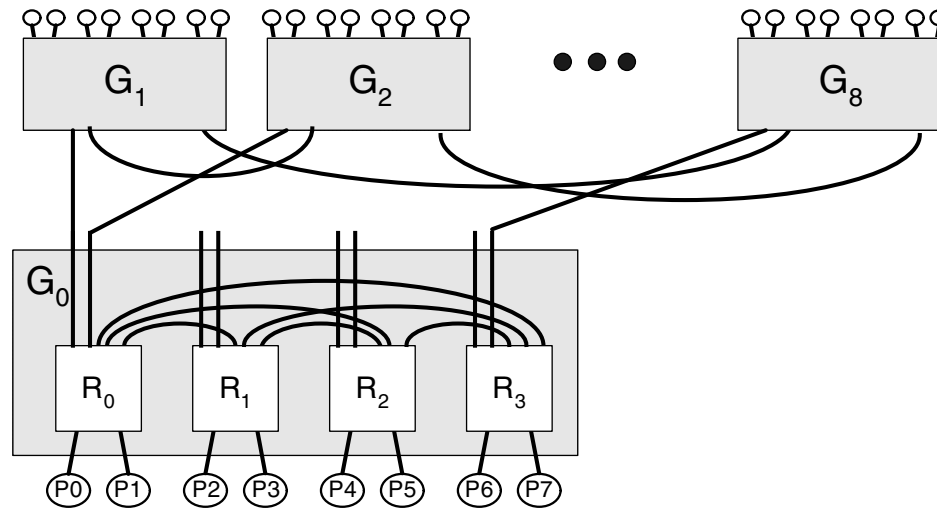
Dragonfly parameters

- p = number of nodes connected to a switch
- a = number of switches in a group
- h = number of optical links on a switch



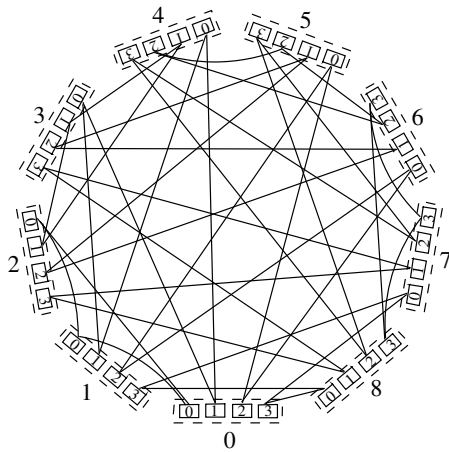
- Number of groups $g = ah+1$

Which port connects to which group?

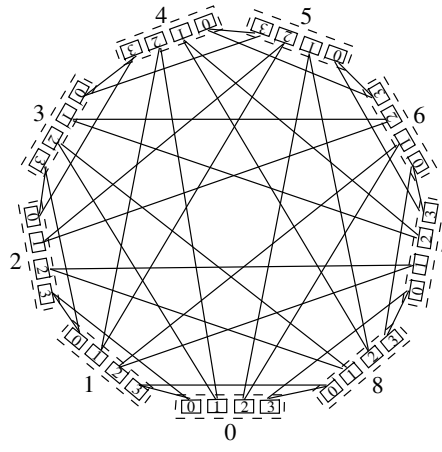


From original Dragonfly paper: Kim et al., ISCA 2008

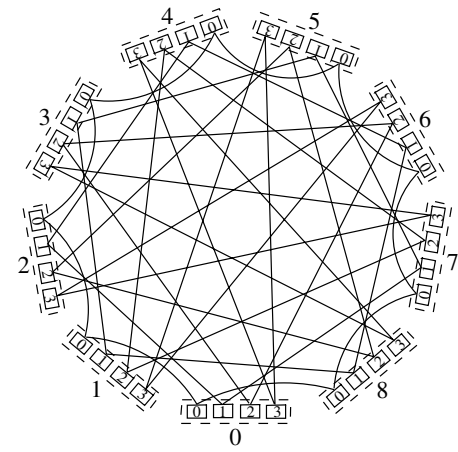
Three distinct global link arrangements



Absolute arrangement



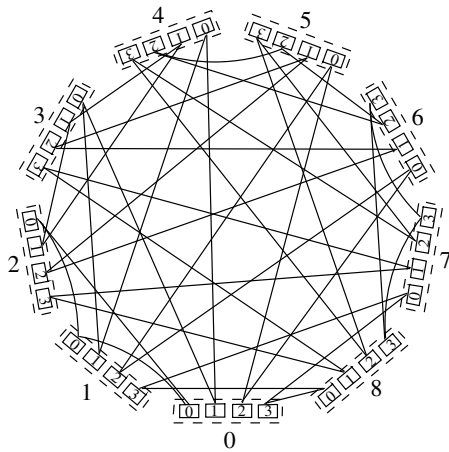
Relative arrangement



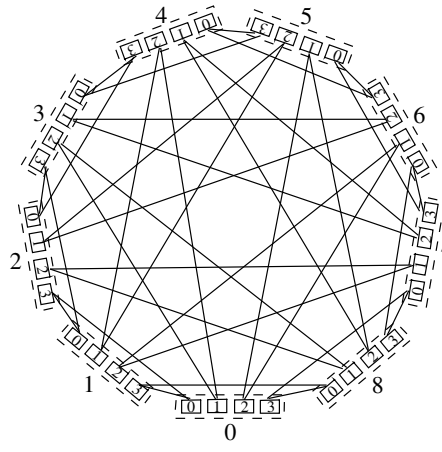
Circulant-based arrangement

Arrangements defined in Camarero et al. ACM Trans. Architect. Code Optim., 2014.

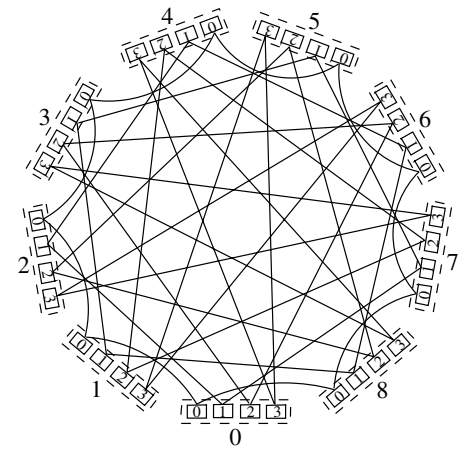
Three distinct global link arrangements



Absolute arrangement



Relative arrangement



Circulant-based arrangement

Arrangements defined in Camarero et al. ACM Trans. Architect. Code Optim., 2014.

Note:

IBM implementation (PERCS) uses absolute

Researchers who draw entire system in their papers use relative

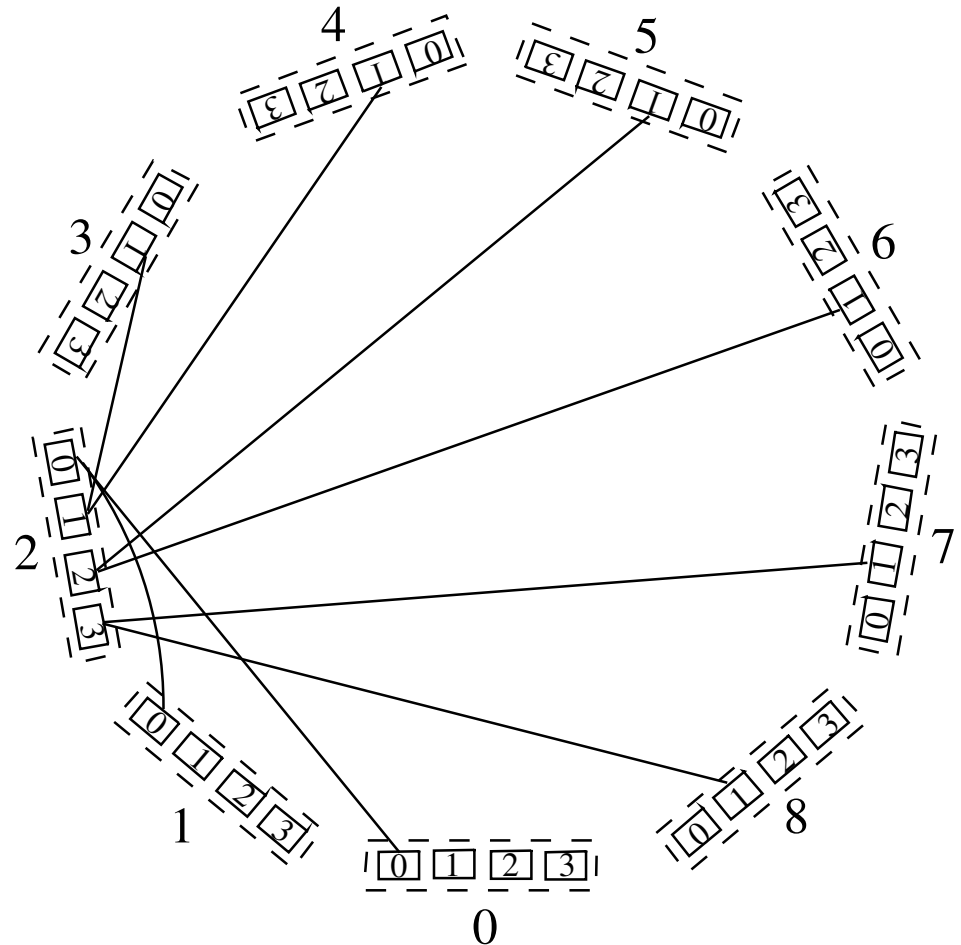
Absolute arrangement

(aka Consecutive arrangement)

**Port k connects to group k
(except skip own group)**

Equivalently, port k of group i
connects to

group k	if $k < i$
group $k+1$	if $k \geq i$

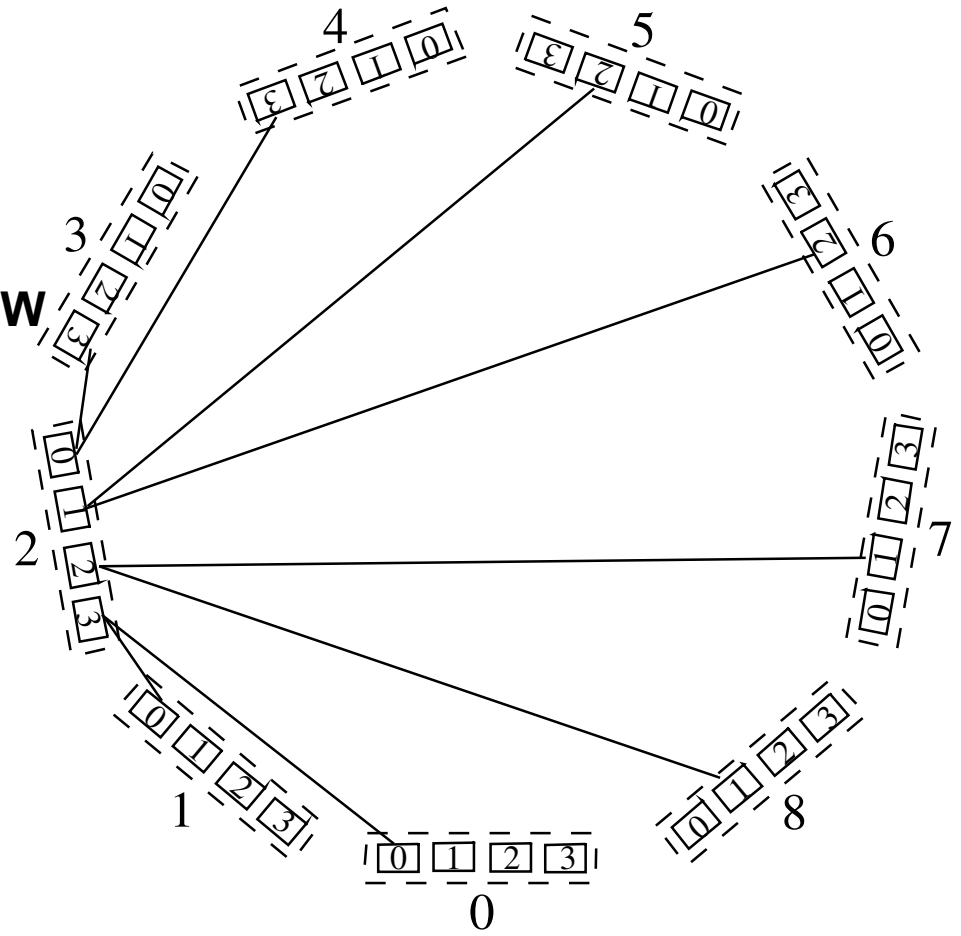


Relative arrangement

(aka Palmtree arrangement)

Port k connects $(k+1)^{\text{st}}$ group CW

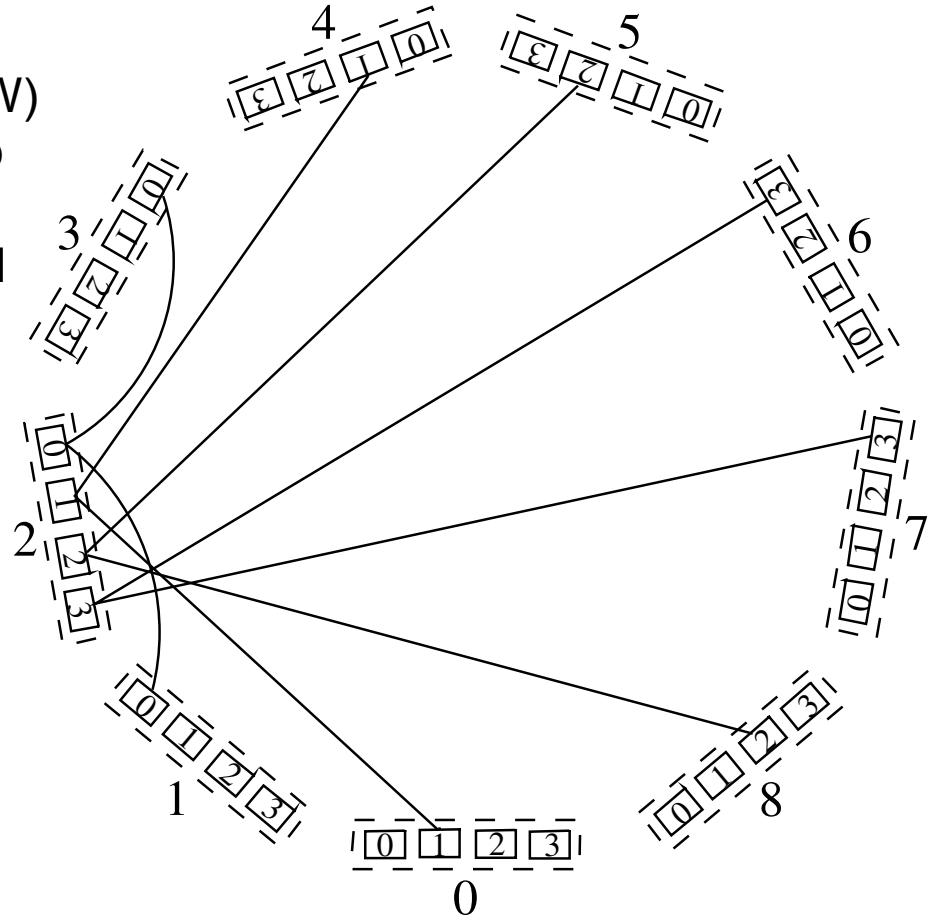
Equivalently, port k of group i
connects to group $(i+k+1) \bmod g$



Circulant-based arrangement

Port 0 connects to next group (CW)
 Port 1 connects to previous group
 Port 2 connects to group 2 ahead
 Port 3 connects to group 2 behind
 ...

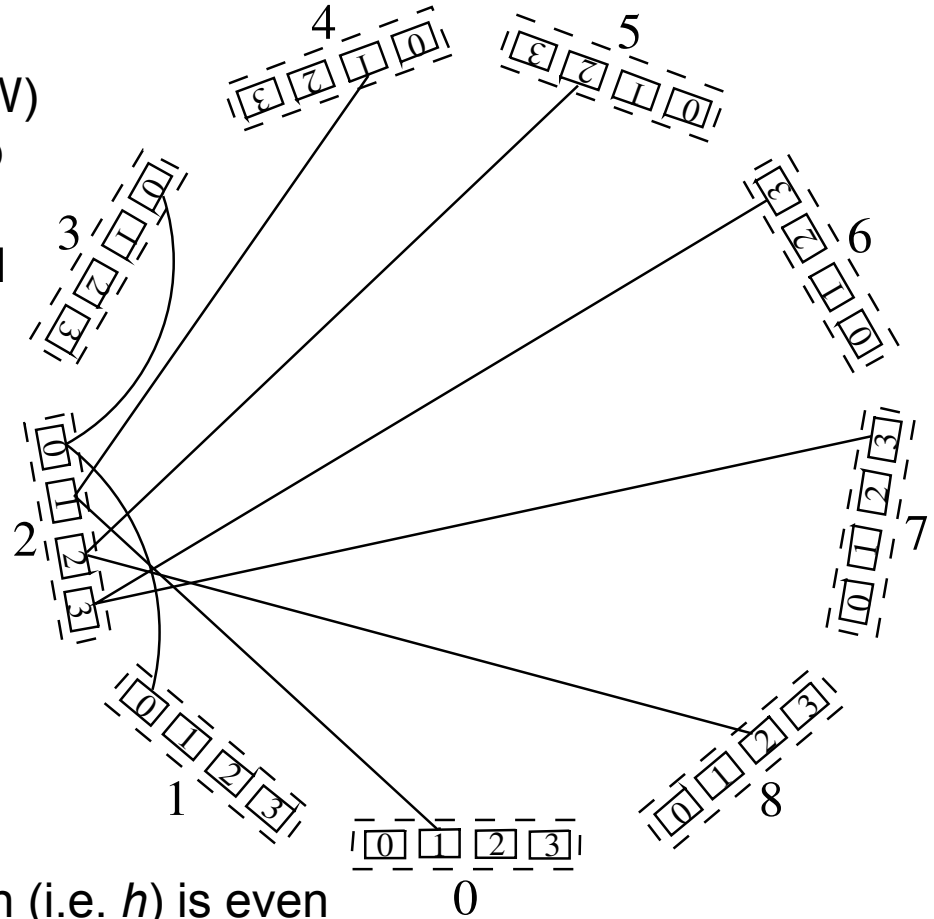
Equivalently, port k of group i
 connects to group
 $(i+k/2+1) \bmod g$ if k is even
 $(i-k/2-1) \bmod g$ if k is odd



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 $(i+k/2+1) \bmod g$ if k is even
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Notes:

Assumes # global links/switch (i.e. h) is even

Always connects switches at same position in their groups

Our contribution

- Comparing absolute, relative, and circulant-based arrangements
 - Bisection bandwidth
 - “Ease of use” with task mapping
 - Criteria for good mapping adapted from Prisacari et al., IPDPS 2013
 - Communication in phases such that
 - Messages distributed evenly on links
 - All paths in a phase have same length

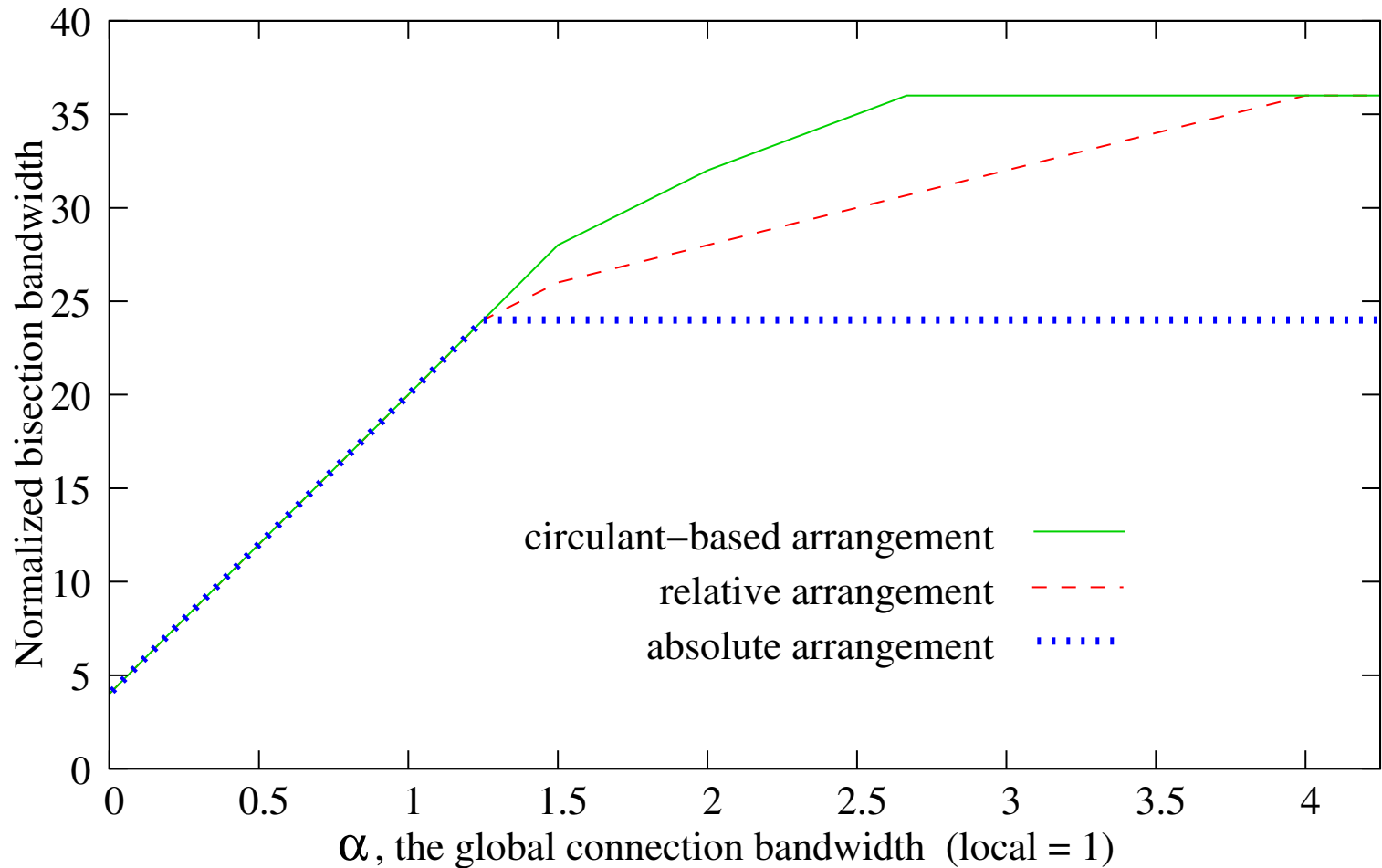
Bisection bandwidth

- Minimum bandwidth between two equal-sized parts of the system
 - Bandwidth for a particular bisection is the (weighted) number of edges crossing from one part to the other
 - Minimize this over all bisections
- Tries to measure worst-case communication bottleneck in a large computation

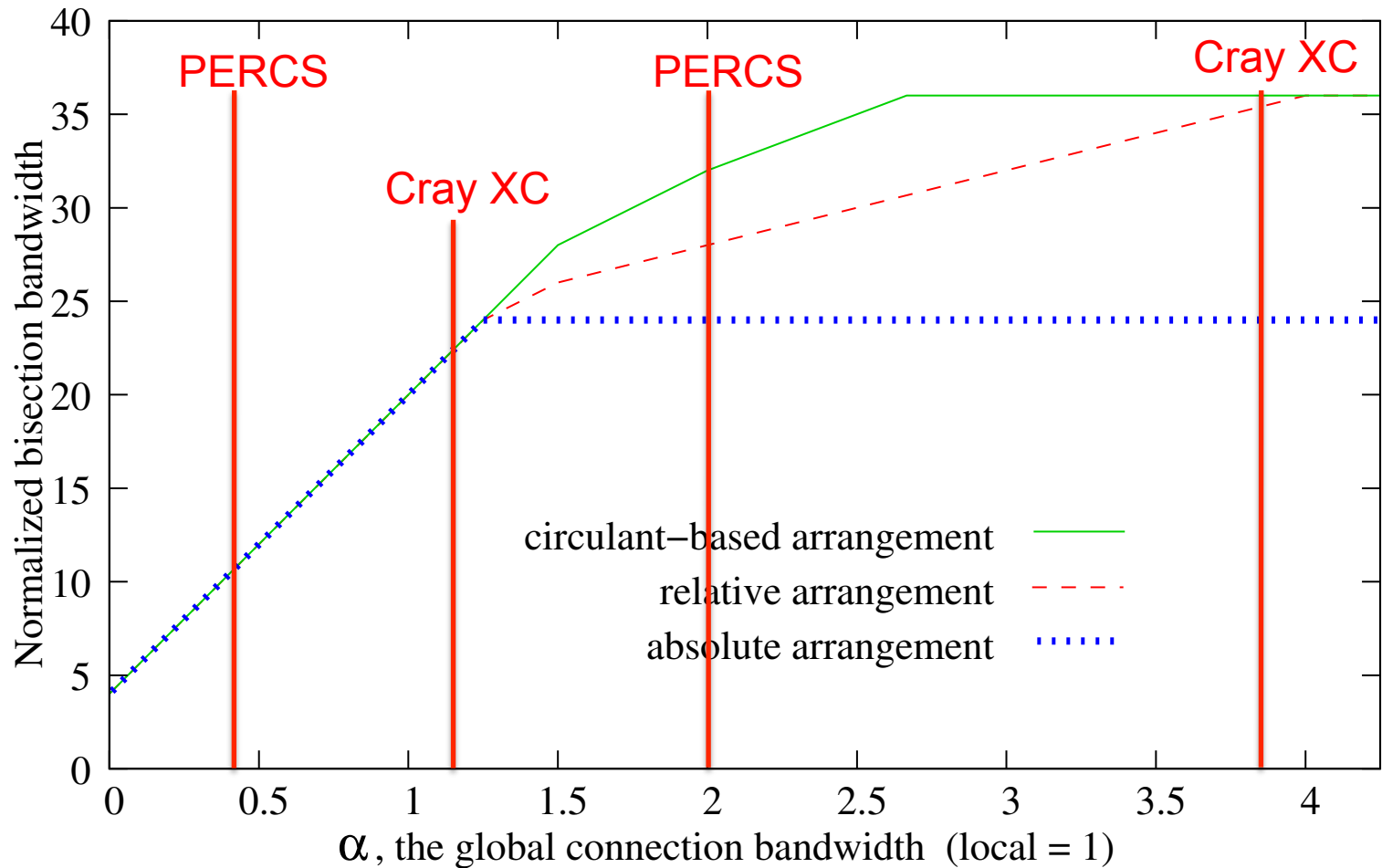
Initial exploration

- Branch and bound on small Dragonfly system (NP-hard ...)
 - $(p,4,2)$: 4 switches per group
 - 2 global links per switch
 - Has 36 switches
- Treat types of edges separately
 - local edges have bandwidth 1
 - global edges have bandwidth α

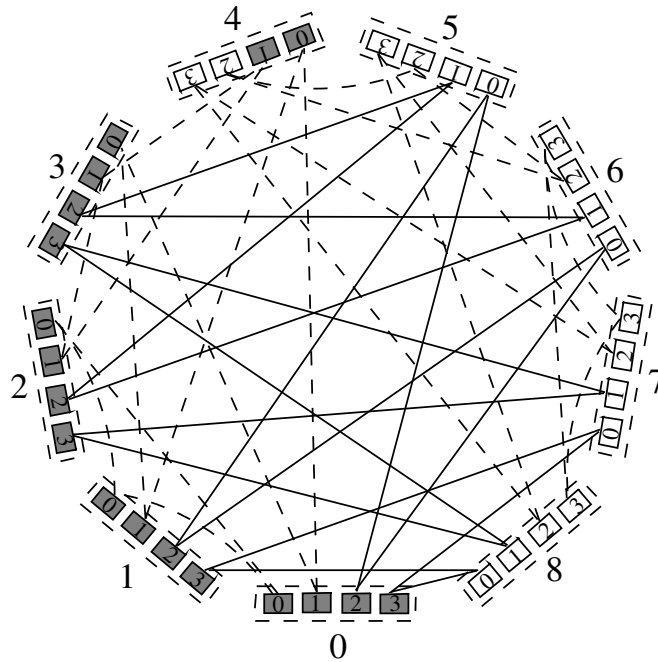
Bisection bandwidth as function of α



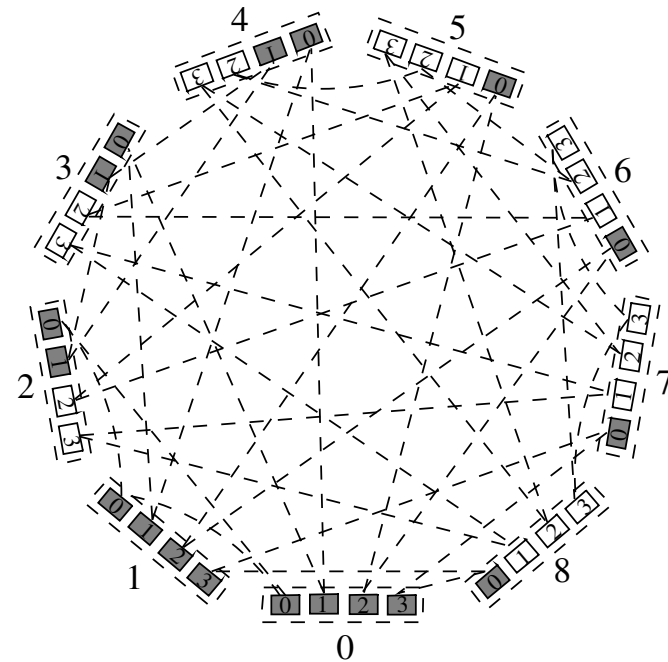
Bisection bandwidth as function of α



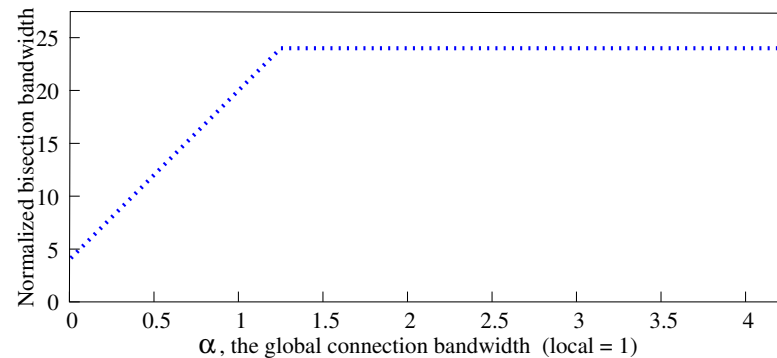
Min-bandwidth cuts for absolute arrangement



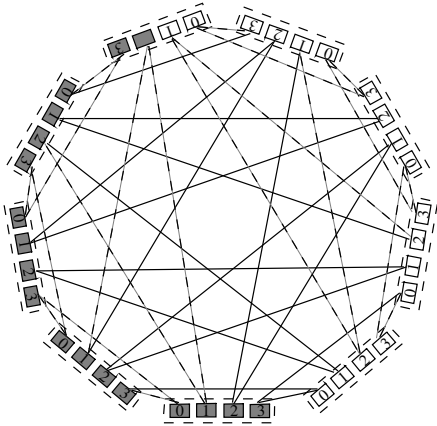
Bandwidth $4 + 16\alpha$



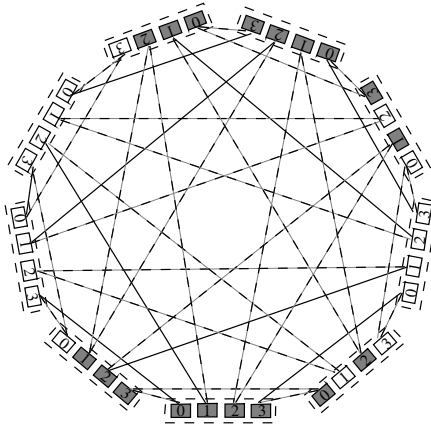
Bandwidth 24



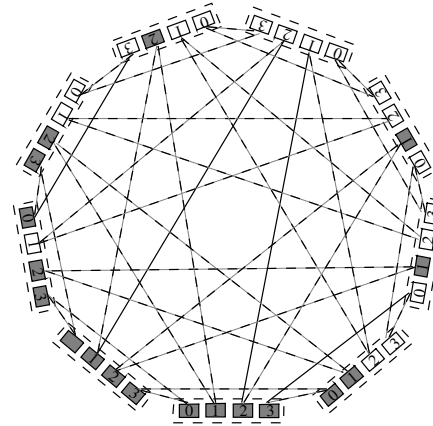
Min-bandwidth cuts for relative arrangement



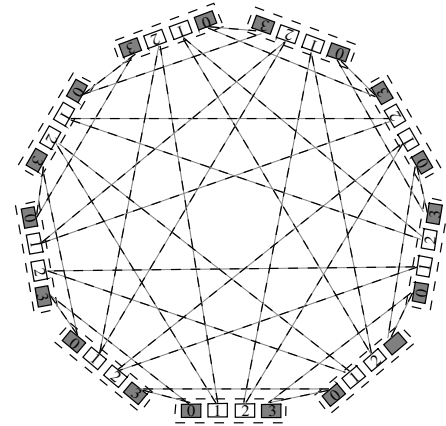
bandwidth $4 + 16\alpha$



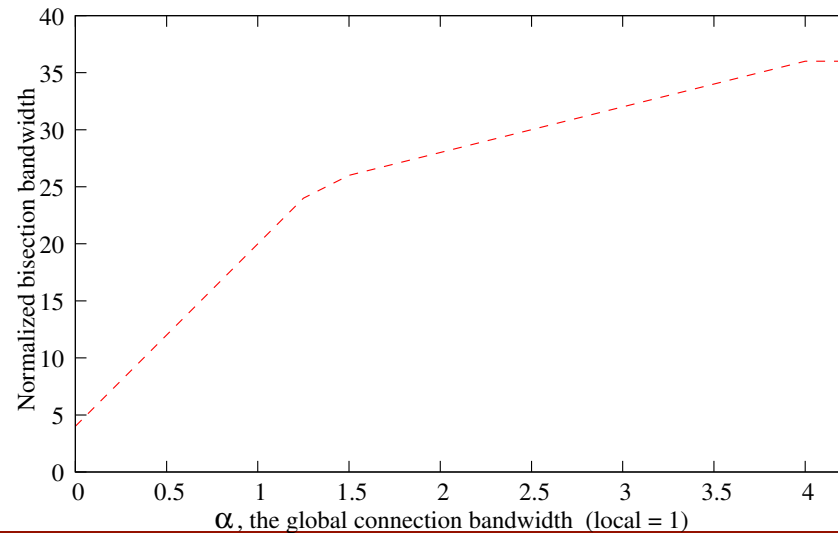
bandwidth $14 + 8\alpha$



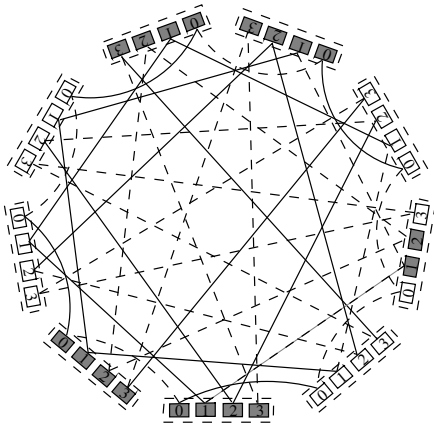
bandwidth $20 + 4\alpha$



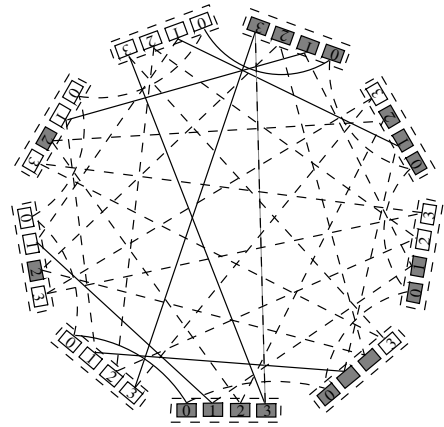
bandwidth 36



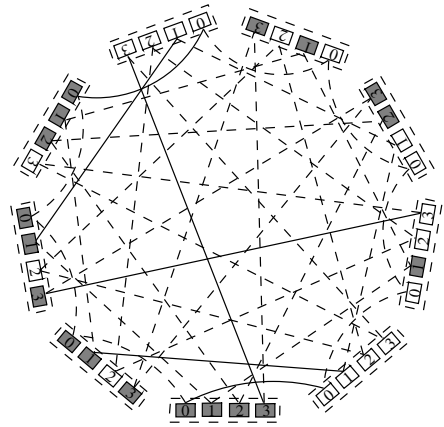
Min-bandwidth cuts for circulant-based arrangement



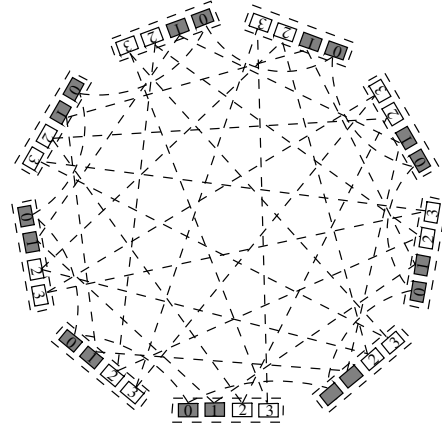
bandwidth $4 + 16\alpha$



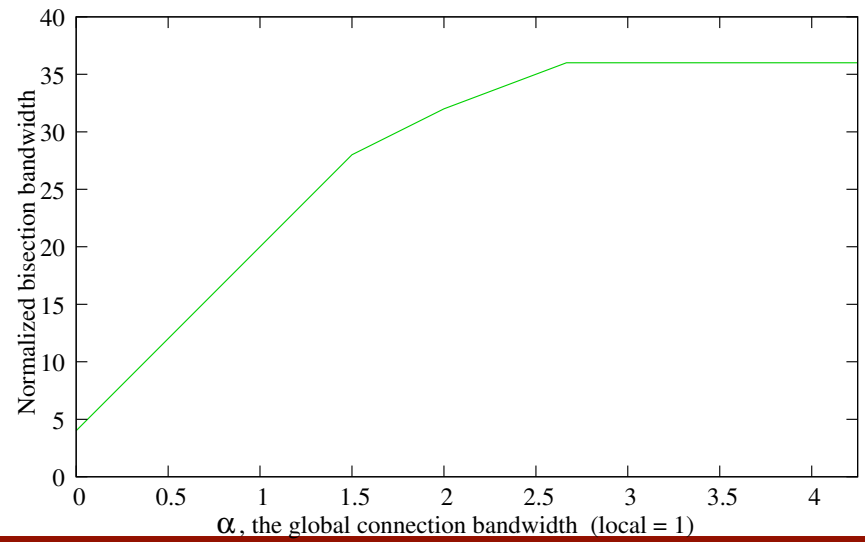
bandwidth $16 + 8\alpha$



bandwidth $20 + 6\alpha$



bandwidth 36



α , the global connection bandwidth (local = 1)

Observations from $(p,4,2)$

- In terms of bisection bandwidth:
Absolute \leq Relative \leq Circulant-based
- For all three arrangements, maximum bisection bandwidth is bounded

Larger networks

- Focus on large α
 - Determine when bisection bandwidth is ultimately limited by local edges
- Globally Connected Component (GCC): Switches that form connected component in graph without local edges

GCCs in Circulant-based arrangements

Recall: Every edge connects two switches at same position in their respective groups

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($a = \text{\#switches/group}$)

GCCs in Circulant-based arrangements

Recall: Every edge connects two switches at same position in their respective groups

There are at least a GCCs $(a = \text{\#switches/group})$

If a is even and α is sufficiently large, the bisection bandwidth is $(a/2)^2g$ $(g = \text{\#groups})$

GCCs in Circulant-based arrangements

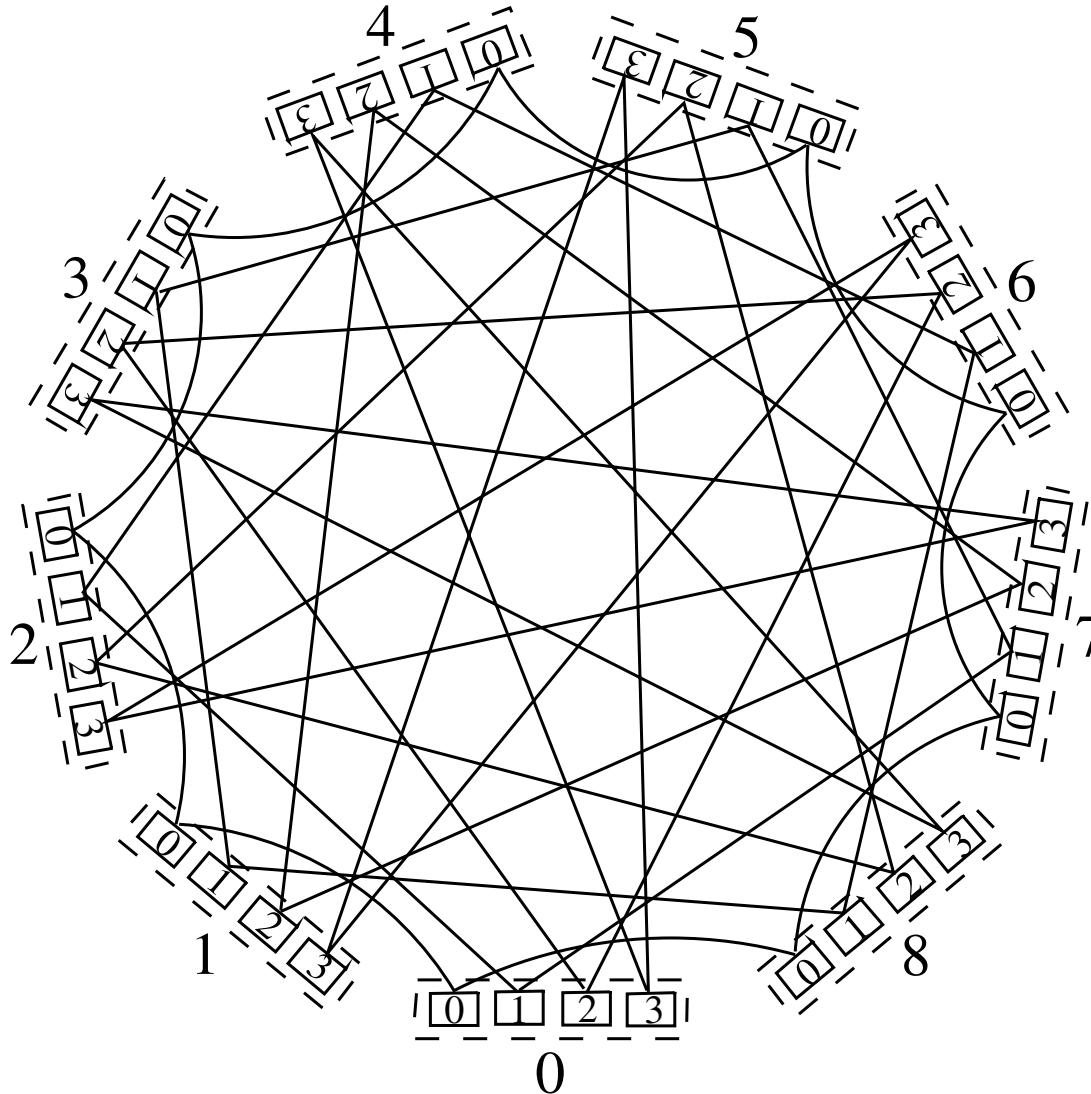
Recall: Every edge connects two switches at same position in their respective groups

There are at least a GCCs ($a = \text{\#switches/group}$)

If a is even and α is sufficiently large, the bisection bandwidth is $(a/2)^2g$ ($g = \text{\#groups}$)

Structure of GCCs potentially more complicated than that, single switch number can be split into multiple GCCs if g is multiple of distance traversed by switch's links

Circulant-based arrangement



GCCs in Relative arrangements

Recall: Port k connects to $(k+1)^{\text{st}}$ group CW

GCCs in Relative arrangements

Recall: Port k connects to $(k+1)^{\text{st}}$ group CW

Switch 0 connects to switch $(\alpha-1)$ in next group

GCCs in Relative arrangements

Recall: Port k connects to $(k+1)^{\text{st}}$ group CW

Switch 0 connects to switch $(a-1)$ in next ~~group~~

h groups

GCCs in Relative arrangements

Recall: Port k connects to $(k+1)^{\text{st}}$ group CW

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h groups

All 0^{th} and $(a-1)^{\text{st}}$ switches form a GCC

GCCs in Relative arrangements

Recall: Port k connects to $(k+1)^{\text{st}}$ group CW

Switch 0 connects to switch $(a-1)$ in next ~~group~~

h groups

All 0^{th} and $(a-1)^{\text{st}}$ switches form a GCC

Generalizes:

$a/2$ GCCs of size $2g$ (plus 1 of size g if a is odd)

Bisection bandwidth in Relative arrangement

When α is sufficiently large, bisection bandwidth is

$(a/2)^2g$ if a is a multiple of 4

$\theta(\alpha)$ otherwise

GCCs in Absolute arrangements

Recall: Port k connects to group k (skip own group)

Gives

$a(a-1)/2$ GCCs of size $2h$

a GCCs of size $h+1$

GCCs in Absolute arrangements

Recall: Port k connects to group k (skip own group)

Gives

$a(a-1)/2$ GCCs of size $2h$

a GCCs of size $h+1$

If a is a multiple of 4, bisection bandwidth is $\leq (a/2)^2g$.

(Also 3 other times, including when $h \leq a/2$)

GCCs in Absolute arrangements

Recall: Port k connects to group k (skip own group)

Gives

$a(a-1)/2$ GCCs of size $2h$

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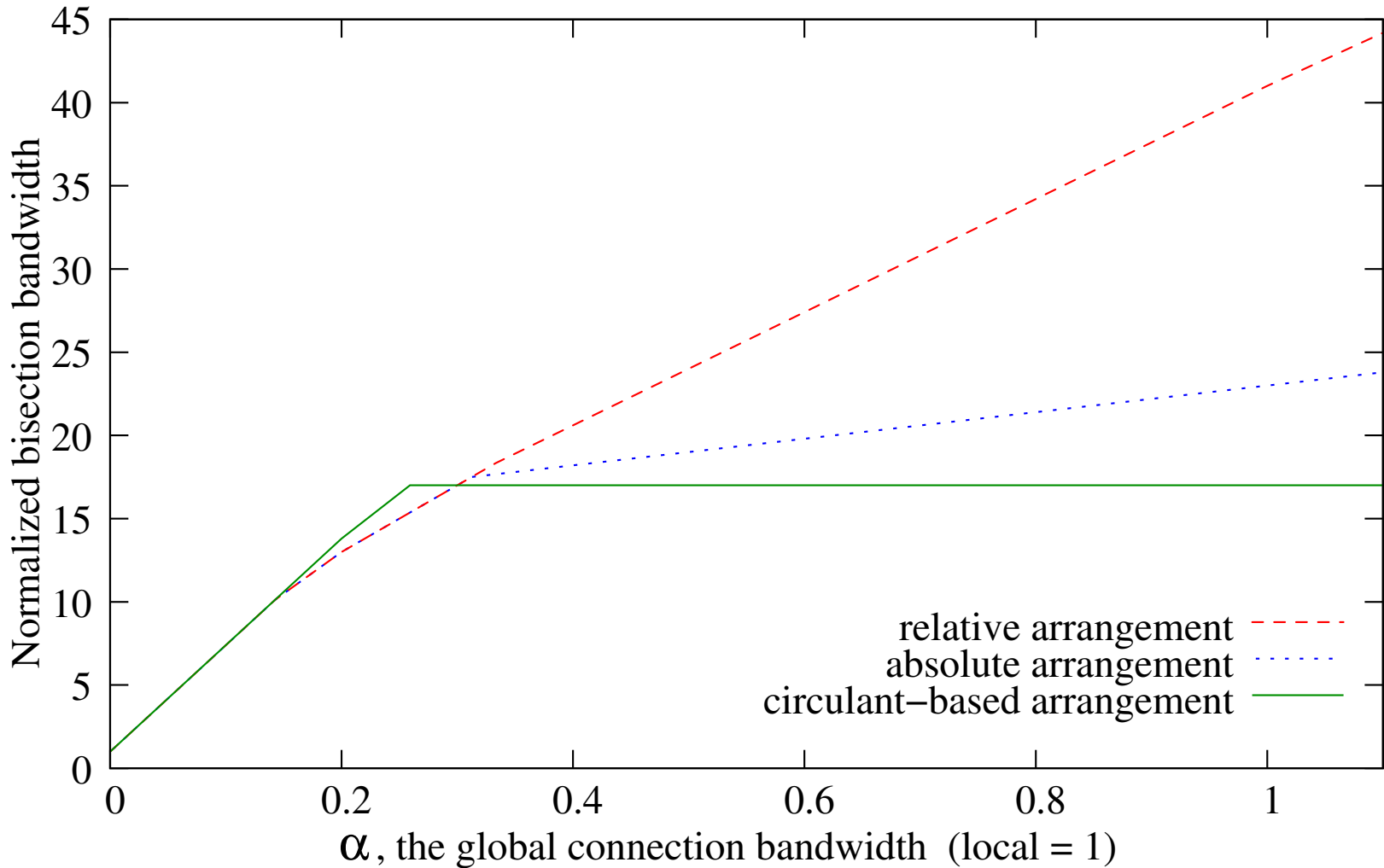
(Also 3 other times, including when $h \leq a/2$)

Otherwise, $\theta(\alpha)$

When bisection bandwidth is bounded

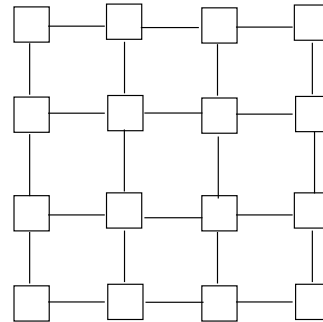
- Circulant: a is even (& other times)
- Relative: a is a multiple of 4
- Absolute: a is a multiple of 4 (& 3 other times, including when $h \leq a/2$)

Normalize bisection bandwidth for ($p, 2, 8$)



Task mapping

- Assignment of tasks to compute nodes to minimize contention



- Our assumptions:

- Stencil jobs

- Tasks blocked to fit on entire switch

Criteria for good mapping

Adapted from Prisacari et al., IPDPS 2013:

Communication in phases such that:

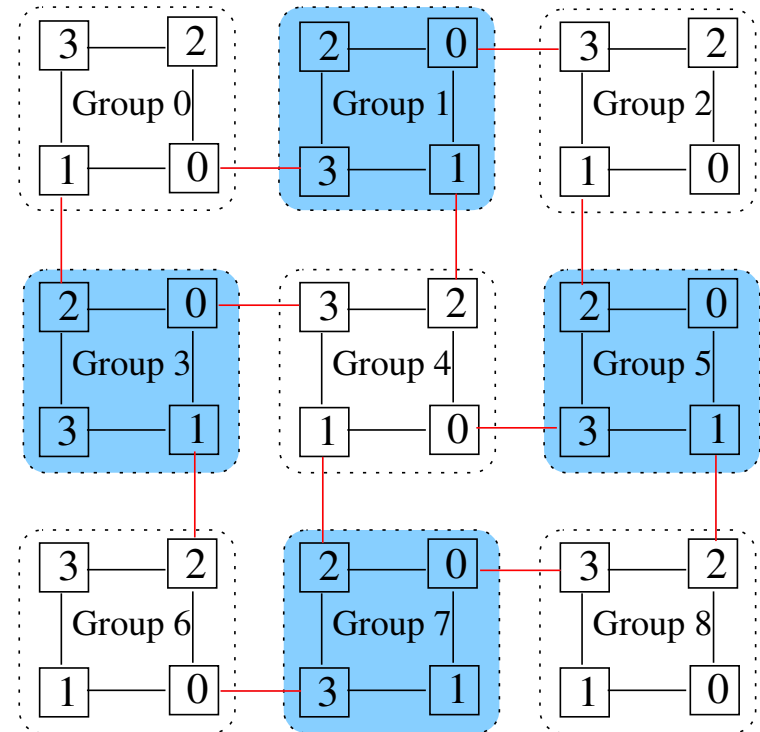
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Mapping of 6×6 job onto $(p,4,2)$ Dragonfly
with relative global link arrangement

Criteria for good mapping

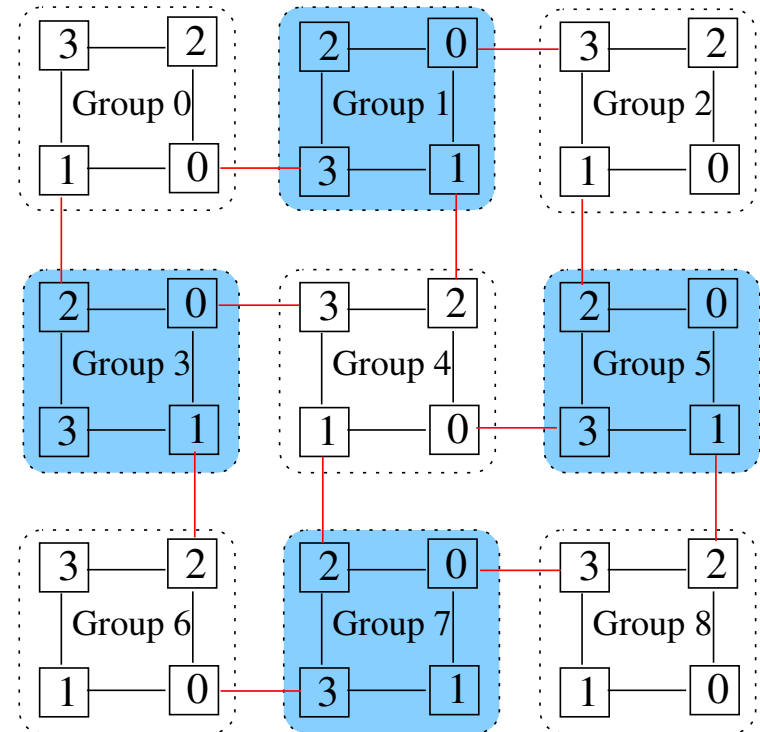
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Communication in phases such that:

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Phases for this mapping:

- Neighbors w/ local links
- Neighbors directly connected by global link
- Neighbors with multi-hop path



Mapping of 6×6 job onto $(p,4,2)$ Dragonfly with relative global link arrangement

Criteria for good mapping

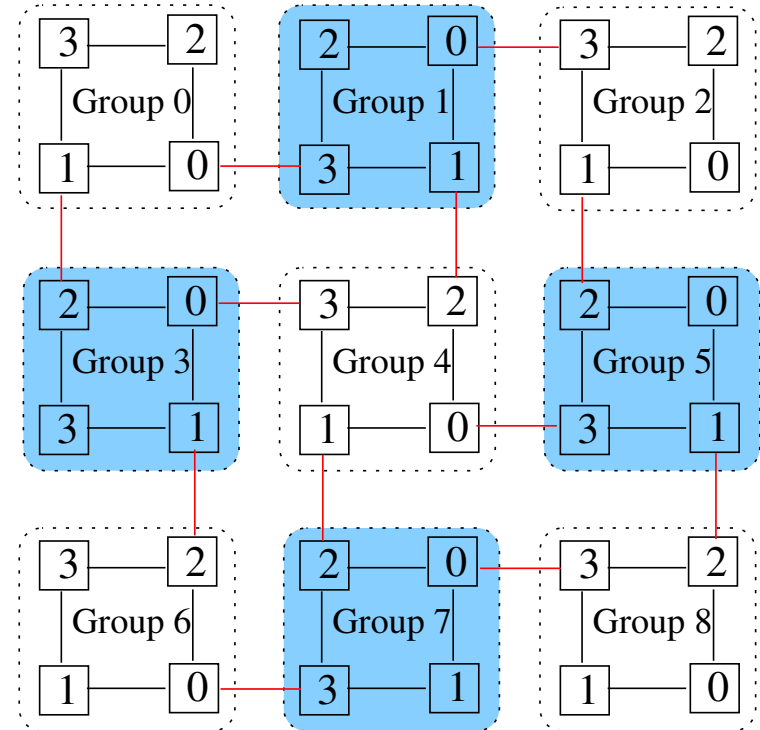
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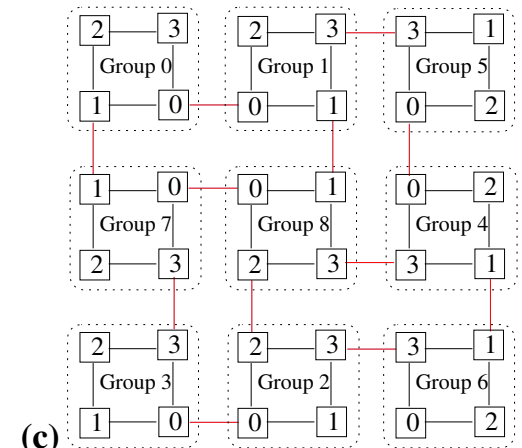
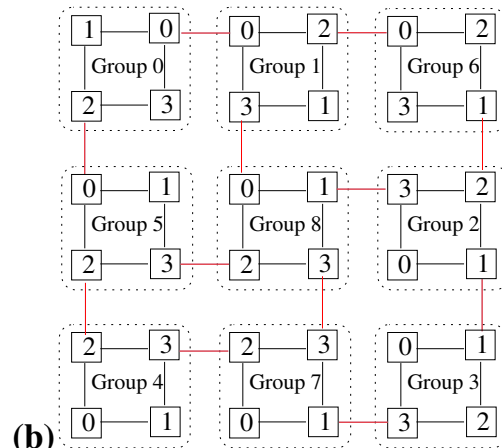
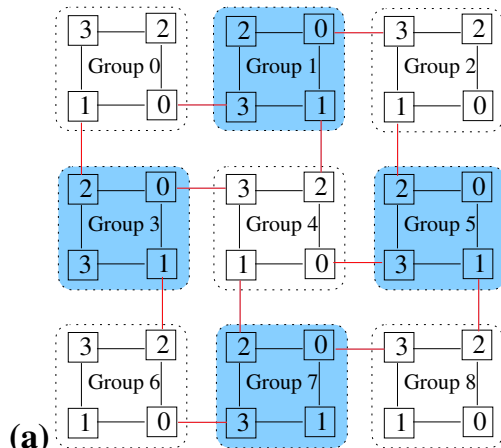
- Neighbors w/ local links
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Mapping of 6×6 job onto $(p,4,2)$ Dragonfly
with relative global link arrangement

Nothing this regular seems to exist for absolute or circulant-based arrangements

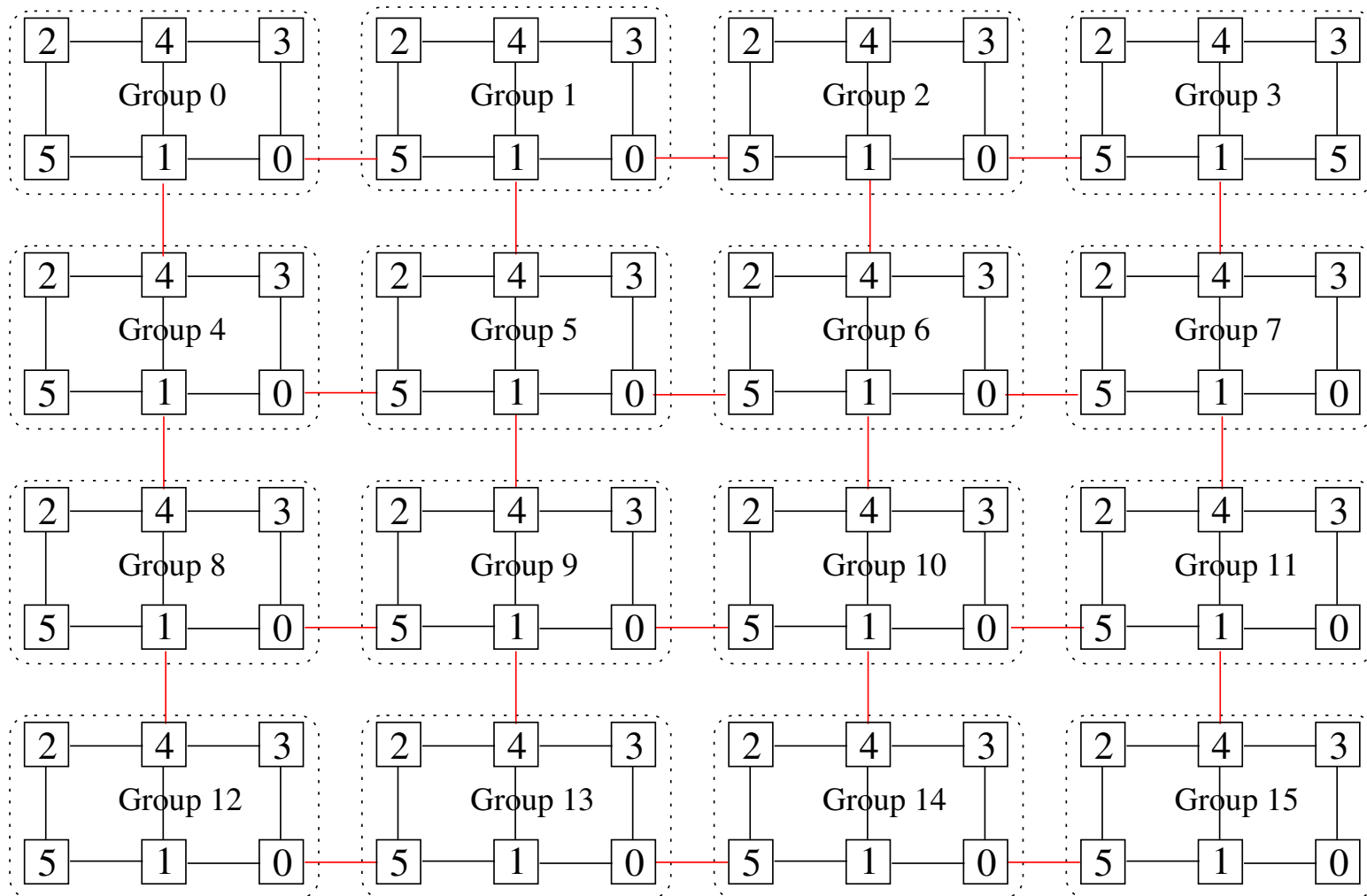
Three mappings for a 6 x 6 stencil job



Conclusions

- On original $(p, 4, 2)$ graph, for bisection bandwidth:
Absolute \leq Relative \leq Circulant-based
- On large graphs, Circulant-based is most often bounded, then Absolute, then Relative
- On $(p, 2, 8)$ graph, at large α :
Circulant-based \leq Absolute \leq Relative
and Absolute and Relative unbounded
- For mapping stencils, Relative gives much more regular mappings

Mapping for a 12 x 8 stencil job on 16 groups of $(p, 6, 3)$ -Dragonfly with rel.



Future work

- Bisection bandwidth at smaller values of α
- Other global link arrangements
- Generalize task mapping and evaluation by simulation
- Communication scheduling recommended by Prisacari et al. may be difficult to implement
- Early Sandia Trinity applications measurements
 - Communications stalls surprisingly high
 - Thermal problems in turbo mode, 25°F swings

Thanks!

- vjleung@sandia.gov