Malleable task-graph scheduling with a practical speed-up model

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New Challenges in Scheduling Theory — Aussois

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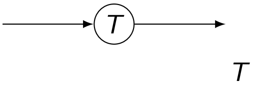
Motivation

Context:

- Optimize the time performance of multifrontal sparse solvers (e.g., MUMPS or QR-MUMPS)
- Computations well described by a tree of tasks
- Generalization to Series-Parallel graphs
- Purpose: find a schedule achieving the lowest makespan



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- Provide theoretical guarantees on widely used scheduling algorithms
- Design ones with smaller makespan

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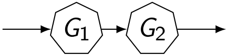

$G_1 ; G_2$

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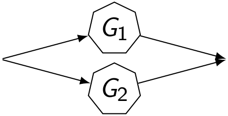

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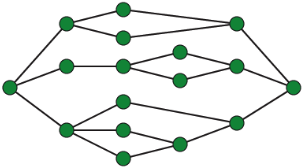

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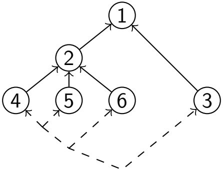

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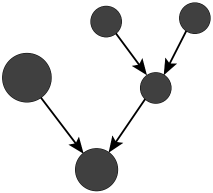
Application modeling

Coarse-grain picture: tree of tasks (or SP task graph)

- Each task: partial factorization, graph of smaller sub-tasks



- 😕 Expand all tasks and schedule resulting graph ?
- 🙂 Scheduling trees simpler than general graphs (forget sub-tasks)

Behavior of coarse-grain tasks

- parallel and malleable
- Speed-up model ⟶ trade-off between:
  - Accuracy : fits well the data
  - Tractability : amenable to perf. analysis, guaranteed algorithms

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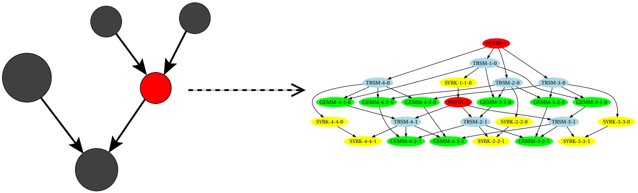

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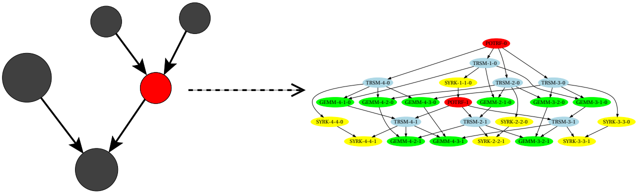

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General speed-up models

Literature: studies with few assumptions

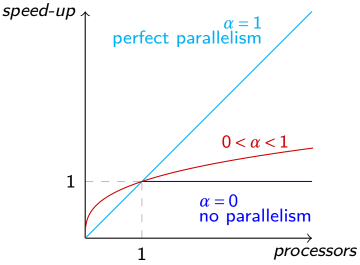
$$speed\text{-}up(p) = \frac{time(1\ proc.)}{time(p\ proc.)} \quad \Big| \quad work(p) = p \cdot time(p\ proc.)$$

Non-increasing speed-up and work

- Independent tasks: theoretical FPTAS and practical 2-approximations [Jansen 2004, Fan et al. 2012]
- SP-graphs: ≈ 2.6-approximation [Lepère et al. 2001] with concave speed-up: $(2+\varepsilon)$-approximation of unspecified complexity [Makarychev et al. 2014]

Previous work (Europar 2015, with A. Guermouche)

Prasanna & Musicus model [PM 1996]: $\text{speed}-\text{up}(p) = p^{\alpha}$



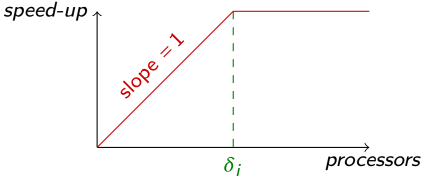
Conclusions:

- Average Accuracy 😐
- Rational numbers of processors 😕
- Optimal algorithm for SP-graphs 🙂
- No guarantees for distributed platforms ☹
- Task finish times complex to compute ☹

Today: simpler model

Simple and reasonable model of a parallel malleable task T_i

- Perfect parallelism up to a threshold δ_i: $time = w_i / \min(p, \delta_i)$
- Rational allocation for free (McNaughton's wrap-around rule) ☺



Related studies

- 2-approximation [Balmin et al. 13] that we will discuss
- [Kell et al. 2015]: $time = \frac{w_i}{p} + (p-1)c$;
  2-approximation for $p = 3$, open for $p \geq 4$

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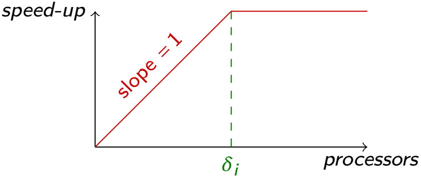

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Outline

1. Problem complexity
2. Analysis of ProportionalMapping [Pothen et al. 1993]
3. Design of a greedy strategy
4. Experimental comparison
5. Conclusion

Overview of the problem

Given a SP-graph, p processors: compute the optimal makespan

- Problem known as $P|sp-graph, any, spdp\text{-}lin, \delta_i|C_{\max}$
- Malleability + perfect parallelism $\Longrightarrow$ P ☺
- ... + thresholds $\Longrightarrow$ NP-complete ☹
- Existing proof in [Drozdowski and Kubiak 1999] : arguably complex

Contribution

- New NP-completeness proof

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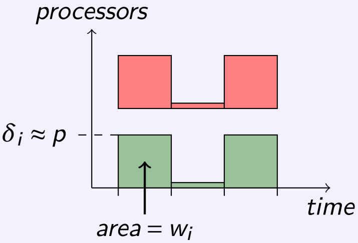
Widget for the proof

Two 3-task chains



Each task:

- $\delta_i = w_i$
- min. computing time of 1

Simultaneous start: $C_{max} \approx 5$

Time-shift: $C_{max} \approx 4$

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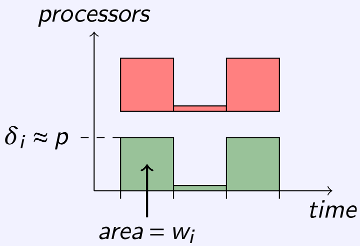

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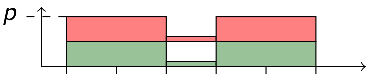

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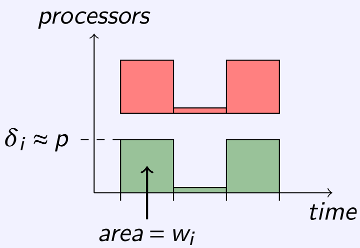

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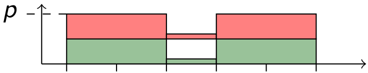

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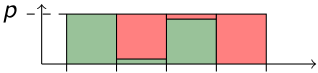
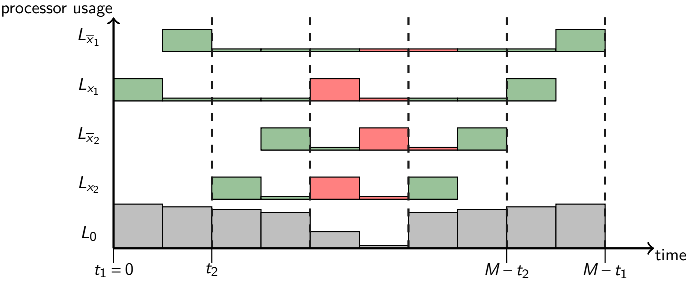


Proof sketch

Reduction from 3-SAT (ex: x_1 *OR* x_2 *OR* $\overline{x}_2$)

- Idea: each variable ⇒ a modified widget (a chain for both x_i, $\overline{x}_i$)
- extremities length ⇒ variable — middle ⇒ clause
- The one starting later: TRUE
- Gray chain: profile allowing only *correct* behaviors



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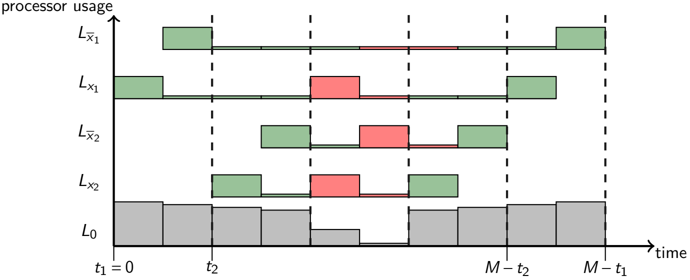

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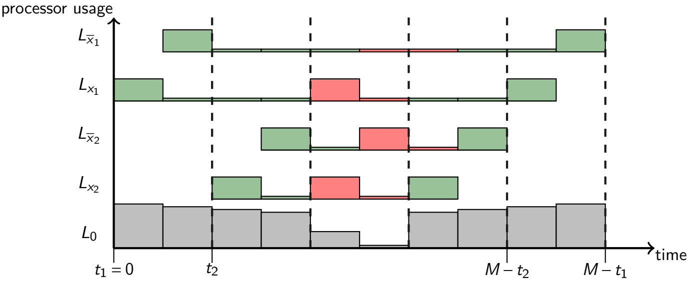

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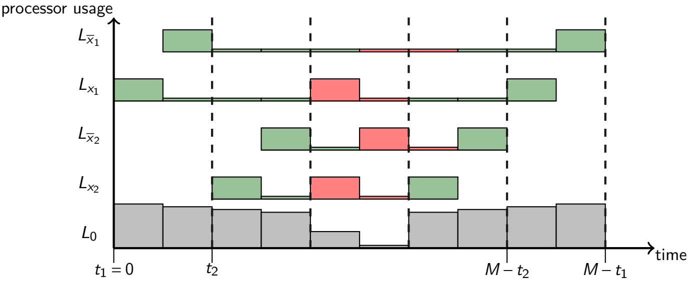

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2. Analysis of ProportionalMapping [Pothen et al. 1993]
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ProportionalMapping [Pothen et al. 1993]

Description

- Simple allocation for trees or SP-graphs
- On $G_1 \parallel G_2$: constant share to G_i, proportional to its weight W_i

Algorithm 1: ProportionalMapping (graph G , q procs)

1 Define the share allocated to sub-graphs of G:

if $G = G_1; G_2; \dots G_k$ **then**
  $\forall i,\ p_i \leftarrow q$

if $G = G_1 \parallel G_2 \parallel \dots G_k$ **then**
  $\forall i,\ p_i \leftarrow q W_i / \sum_j W_j$

2 Call ProportionalMapping (G_i, p_i) for each sub-graph G_i

- Then schedule tasks on p_i processors ASAP

Notes

- Produces a moldable schedule (fixed allocation over time)
- Unaware of task thresholds

Analysis of PROPORTIONALMAPPING schedules

Theorem

PROPORTIONALMAPPING *is a 2-approximation of the optimal makespan.*

Proof.

- Consider makespan without thresholds: $M_\infty \leq M_{\text{opt}}$
- There is an idle-free path Φ from the entry task to the end
- Split the tasks of Φ in two sets:
  - A = tasks limited by their thresholds: $len(A) \leq$ critical path $\leq M_{\text{opt}}$
  - B = tasks limited by the allocation: $len(B) \leq M_\infty \leq M_{\text{opt}}$
- Finally, $M = len(\Phi) = len(A) + len(B) \leq 2M_{\text{opt}}$ □

Note

- Approximation ratio asymptotically tight

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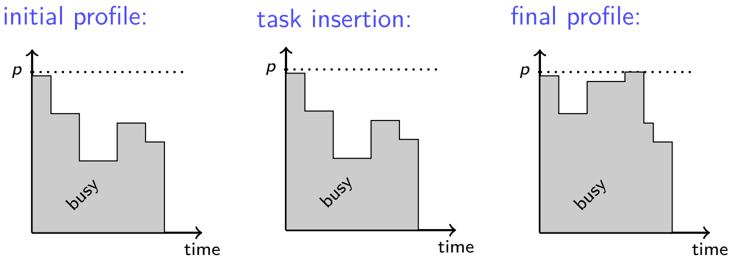
Design of a greedy strategy: GREEDY-FILLING

Algorithm

- Assign priorities to tasks (usually by bottom-level)
- Consider free tasks by decreasing priority
- Greedily insert each task in the current schedule:
  - Compute earliest starting time
  - *Pour* task into the available processor space, respecting thresholds

Illustration



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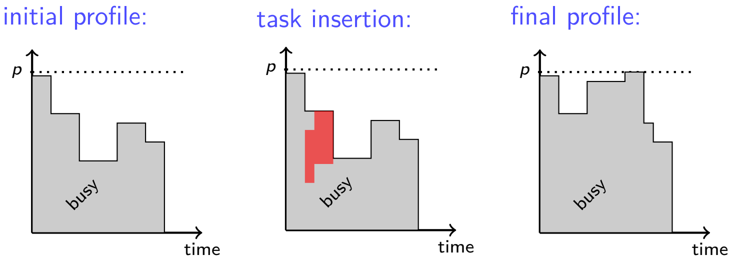

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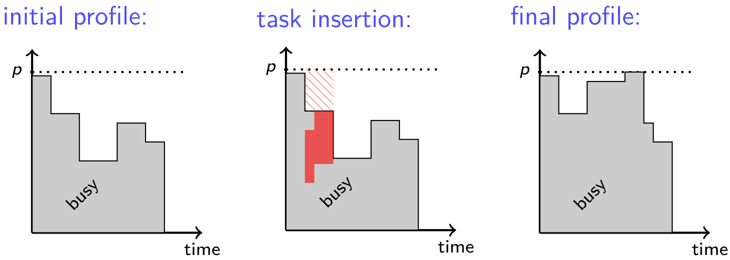

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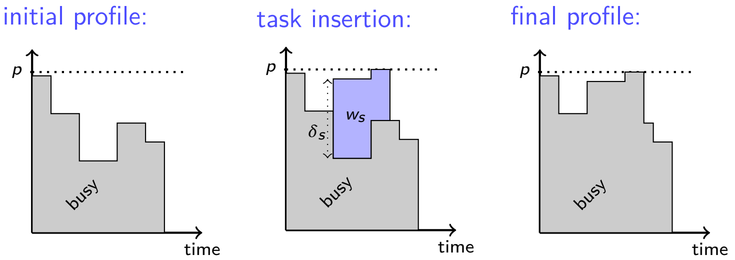

Analysis of Greedy-Filling schedules

Theorem

Greedy-Filling *is a* $2 - \frac{\delta_{\min}}{p}$ *approximation to the optimal makespan.*

Proof.

Transposition of the classical $(2 - \frac{1}{p})$-approximation result by Graham

- Construct a path Φ in G: all idle times happen during tasks of Φ
- Bound *Used* and *Idle* areas ($\mathit{Used} + \mathit{Idle} = p\,M$)
    - At least $\delta_{\min}$ processors busy during Φ

□

Note

- Theorem applies to every strategy without deliberate idle time

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Simulations

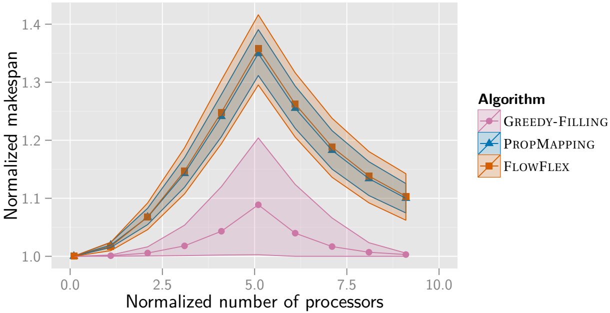
Third algorithm to compare with: FlowFlex

- 2-approximation designed in [Balmin et al. 13] to schedule "Malleable Flows of MapReduce Jobs"
- Solve the problem on an infinite number of processors
- Downscale the allocation on intervals when it is needed

Three datasets

- SYNTH-PROP: Synthetic SP-graphs with $\delta_i = \alpha \times w_i$,
- SYNTH-RAND: Same but with a factor log-uniform in $[0.1\alpha, 10\alpha]$,
- TREES: Assembly trees of sparse matrices, $\delta_i = \alpha \times w_i$.

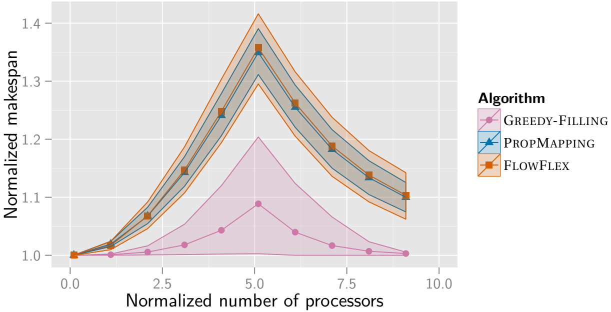
Results on SYNTH-PROP



- Y: Makespan normalized by the lower bound $LB = \max(CP, \frac{W}{p})$
- X: Number of processors normalized by:

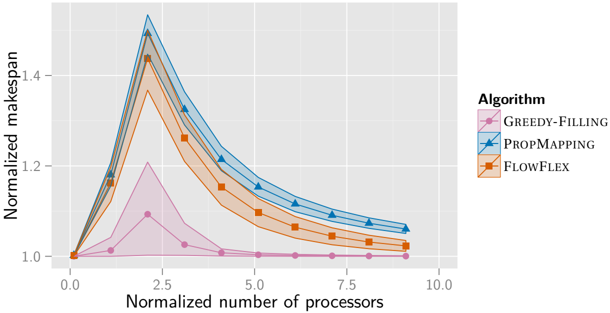
$$parallelism = \frac{\text{makespan with all } \delta_i = 1 \text{ and } p = \infty}{\text{makespan with all } \delta_i = 1 \text{ and } p = 1}$$

Results on SYNTH-PROP



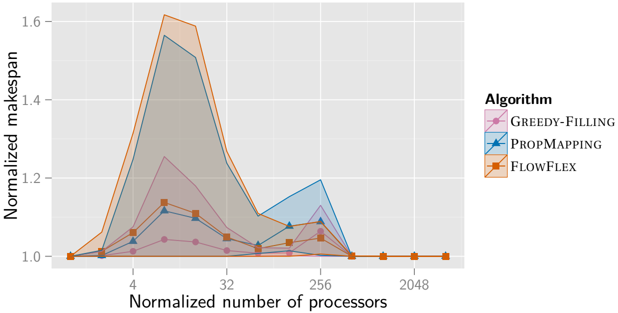
- Plot: mean + ribbon with 90% of the results
- Small/large number of processors: similar results (simpler problem)
- GREEDY-FILLING:
  - ≈ 25% of gain
  - < 20% from the lower bound

Results on SYNTH-RAND



- Similar results with random thresholds
- Larger gaps between GREEDY-FILLING and the others
- Maximum gap happens for smaller platforms

Results on TREES



- Shape of the results depends a lot on the matrix
- Here: one matrix with different ordering and amalgamation parameters
- GREEDY-FILLING (almost always) better than both others
- Smaller maximum gain (around 15%)

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On the algorithms

- PropMapping: does not take advantage of malleability
- FlowFlex: produces gaps that cannot be filled afterwards
- Greedy-Filling: simple, greedy, close to the lower bound

On the model

- Simplest model to account for limited parallelism
- Still NP-complete ☹
- Possible to derive theoretical guarantees (2-approx. algorithms) ☺

Perspectives

- Conduct experiments to assess the model and study thresholds
- Focus on moldable tasks – study the gain of malleability

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On the algorithms

- PropMapping: does not take advantage of malleability
- FlowFlex: produces gaps that cannot be filled afterwards
- Greedy-Filling: simple, greedy, close to the lower bound

On the model

- Simplest model to account for limited parallelism
- Still NP-complete ☹
- Possible to derive theoretical guarantees (2-approx. algorithms) ☺

Perspectives

- Conduct experiments to assess the model and study thresholds
- Focus on moldable tasks – study the gain of malleability