Late Work Scheduling in Online and Offline Mode

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New Challenges in Scheduling Theory – Aussois
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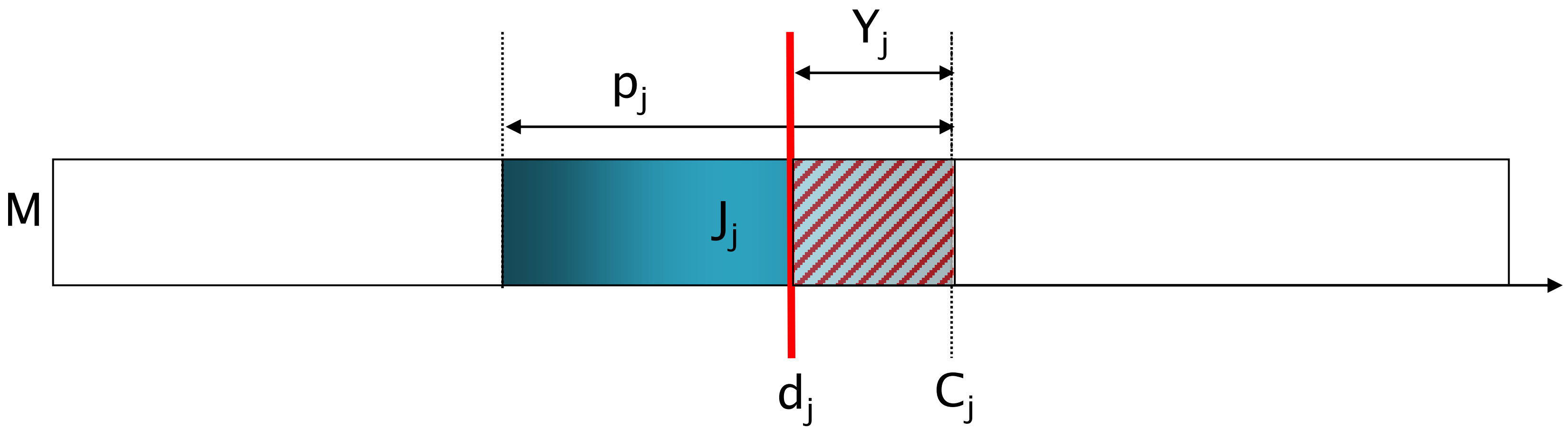
Outline

- Late Work Scheduling
- Problem Definition
- Offline Cases
- Online Cases
- Conclusions & Future Research Directions

Late Work

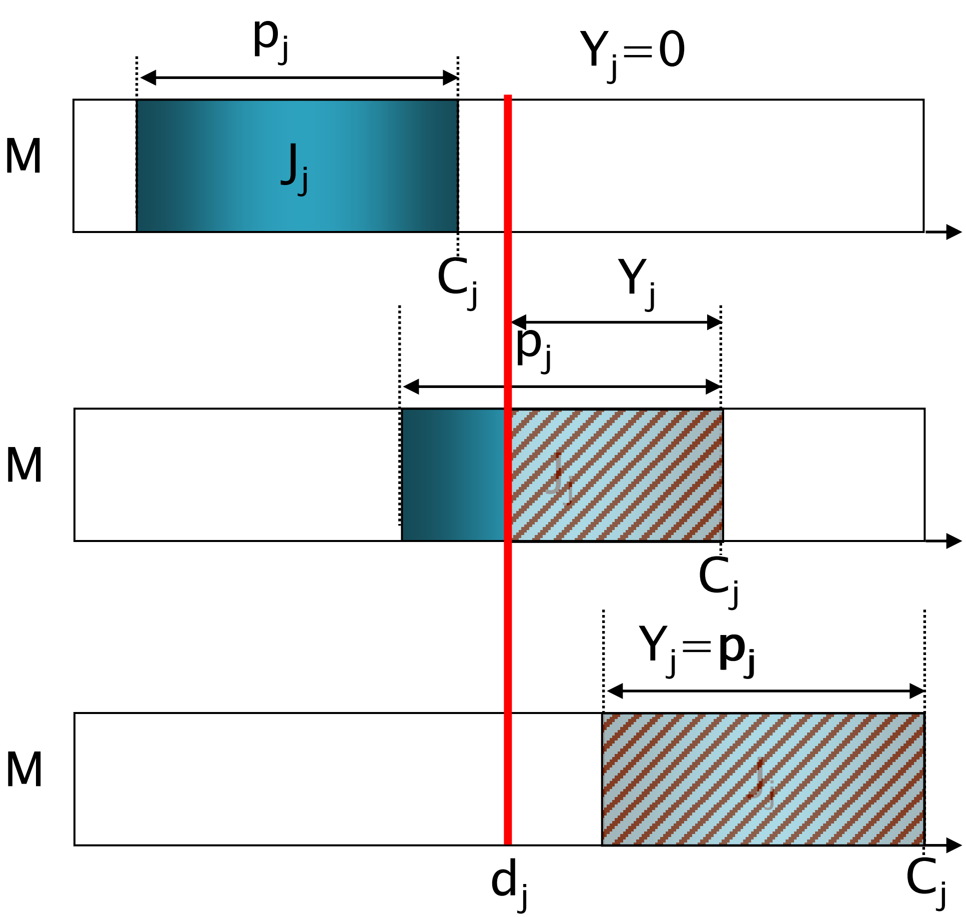
- late work criteria
  minimize the amount of work
  executed after the predefined due date

  (Błażewicz, 1984)



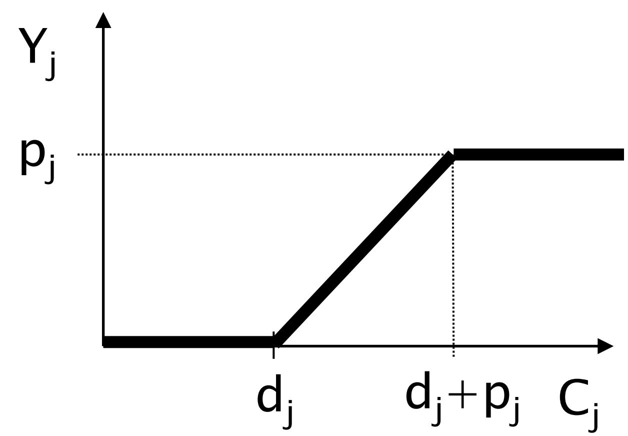
$$Y_j = \min\{\max\{0, C_j - d_j\}, p_j\}$$

Late Work

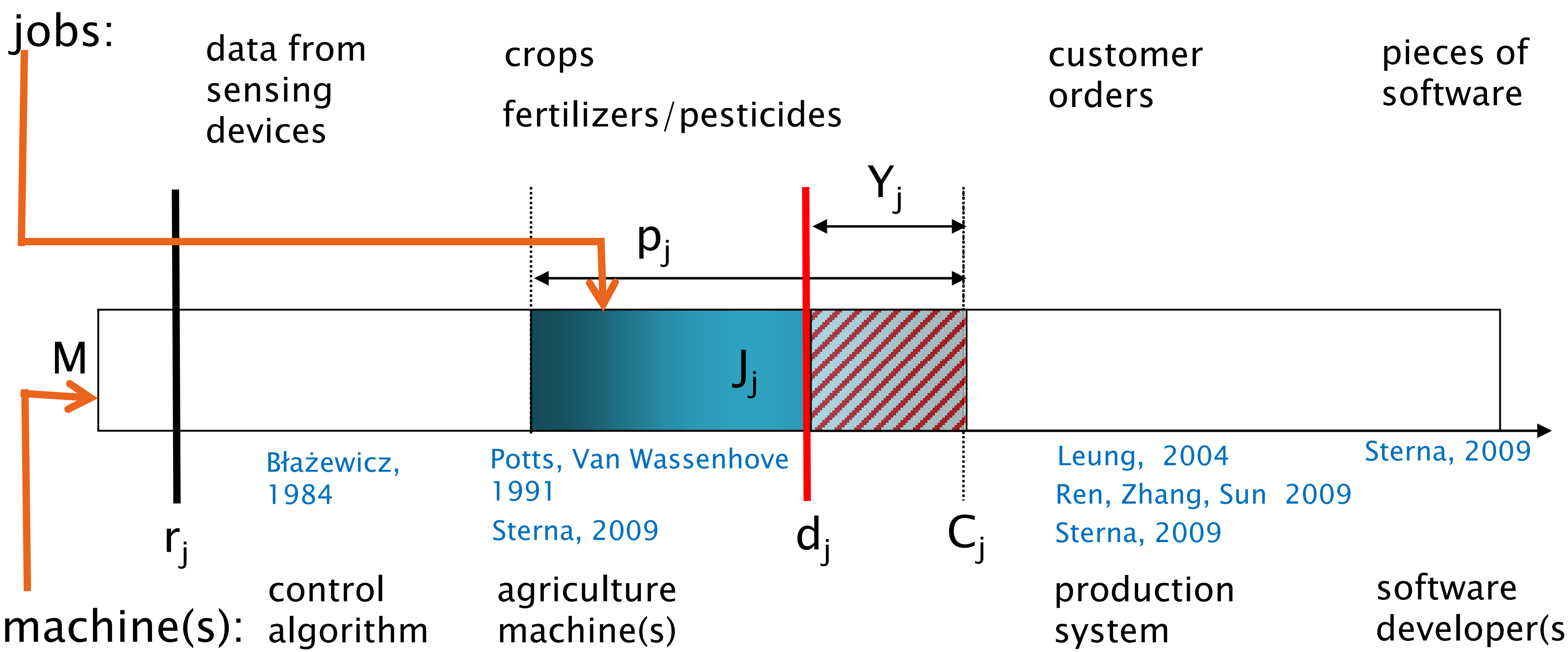


$$Y_j = \min\{\max\{0, C_j - d_j\}, p_j\}$$

$$= \min\{D_j, p_j\}$$



Motivations/Applications



M.Sterna, A survey of scheduling problems with late work criteria, Omega 39 (2011) 120-129

Single Machine Problems

$*$ DP-benevolent problem (Woeginger,2000)

polynomially solvable

$1|pmtn|Y$ (Potts, Van Wassenhove, 1991)

$1|r_j, pmtn|Y$ (Hochbaum, Shamir, 1990) (Lin, Hsu, 2005)

$1|pmtn|Y_w$ (Hariri, Potts, Van Wassenhove, 1995) (Hochbaum, Shamir, 1990)

$1|r_j, pmtn|Y_w$ (Hochbaum, Shamir, 1990) (Yu, 1991) (Leung, Yu, Wei, 1994)

$1|d_j=d|Y$ $1|p_j=p|Y$ (Potts, Van Wassenhove, 1991)

$1|r_j, d_j=d|Y$ (Lin, Hsu, 2005)

$1|d_j=d|Y_w$ $1|p_j=p|Y_w$ (Hariri, Potts, Van Wassenhove, 1995)

$1|r_j, p_j=1|Y_w$ (Sterna, 2000)

NP-hard

$1||Y^*$ (Potts, Van Wassenhove, 1991) (Leung, 2004)

$1|r_j|Y$ (Potts, Van Wassenhove, 1991) (Lin, Hsu, 2005)

$1||Y_w^*$ (Hariri, Potts, Van Wassenhove, 1995) (Kovalyov, Potts, Van Wassenhove, 1995)

$1|B\geq n|Y$ (Zhang and Wang, 2005)

$1|B\geq n|Y_w^*$ (Ren, Zhang, Sun, 2009)

$1|p_j=1,chains|Y$ (Sterna, 2000)

Dedicated Machines Problems

polynomially solvable

$O2|d_j=d|Y$
(Błażewicz, Pesch, Sterna Werner, 2004)

$O|r_j, pmtn|Y_w$
(Błażewicz, Pesch, Sterna Werner, 2004)

NP-hard

$O2|d_j \in \{d_1,d_2\}|Y$ (Leung, 2004)

$F2|d_j=d|Y$ (Lin, Lin, Lee, 2006)
(Sterna, 2007)

$F2|d_j \in \{d_1,d_2\}|Y$ (Leung, 2004)

$F2||Y$ (Leung, 2004)

$F|r_j|Y$ (Pesch, Sterna, 2009)

$O2|d_j=d|Y_w$
(Błażewicz, Pesch, Sterna, Werner, 2004)

$F2|d_j=d|Y_w$
(Błażewicz, Pesch, Sterna, Werner, 2005)

$J2|d_j=d, n_j \le 2|Y_w$
(Błażewicz, Pesch, Sterna, Werner, 2007)

Parallel Machines Problems

polynomially solvable

$P|r_j,pmtn|Y$ (Leung, 2004)

$P|r_j,pmtn|Y_w$ (Błażewicz, 1984)
(Błażewicz, Finke, 1987)

$Q|r_j,pmtn|Y$ (Leung, 2004)

$Qm|r_j,pmtn|Y_w$ (Błażewicz, 1984)

$Q|r_j,pmtn|Y_w$ (Błażewicz, Finke, 1987)
(Leung, 2004)

$Pm|r_j, p_j=1|Y_w$ $P|r_j, p_j=1|Y_w$
(Sterna, 2000)

$Q|p_j=1|Y_w$

NP-hard

$P||Y$ (Błażewicz, 1984)

$P2|p_j=1,chains|Y$

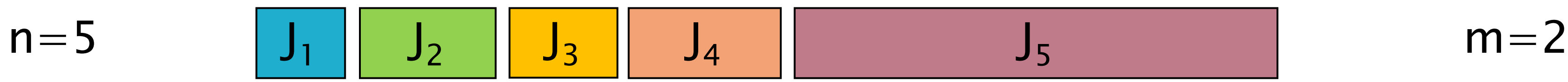
Problem Definition – $P|d_j=d|Y$

- a set of jobs $J=\{J_1, \ldots, J_n\}$
  with processing time p_j for job J_j
- a set of identical parallel machines $M=\{M_1, \ldots, M_m\}$
- common due date d
- criterion: minimizing total late work

- offline case: the set of jobs J is known in advance
- online case: jobs appear „over list"

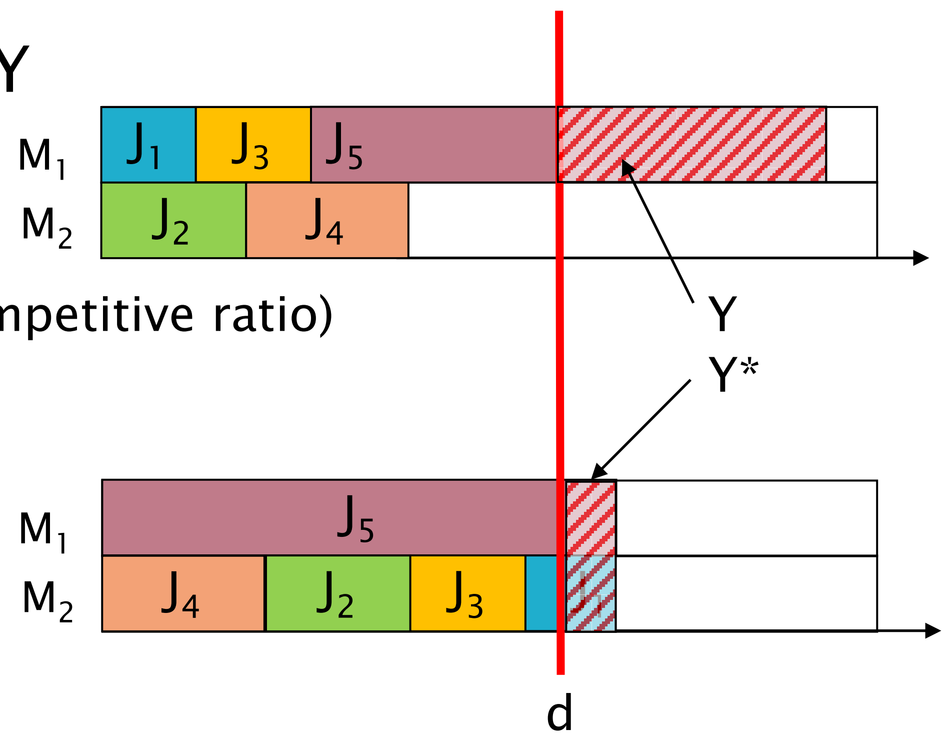
- $P2|d_j=d|Y$ and $P2|d_j=d$, online over list$|Y$
- $P|d_j=d|Y$ and $P|d_j=d$, online over list$|Y$

Problem Definition



- $P2|d_j{=}d$, online over list$|Y$
  - lower bound
  - upper bound

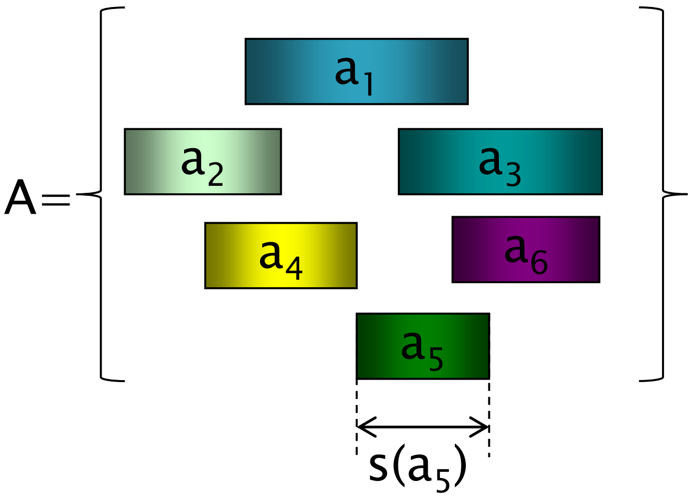
    (online algorithm with finite competitive ratio)
- $P2|d_j{=}d|Y$
  - computational complexity

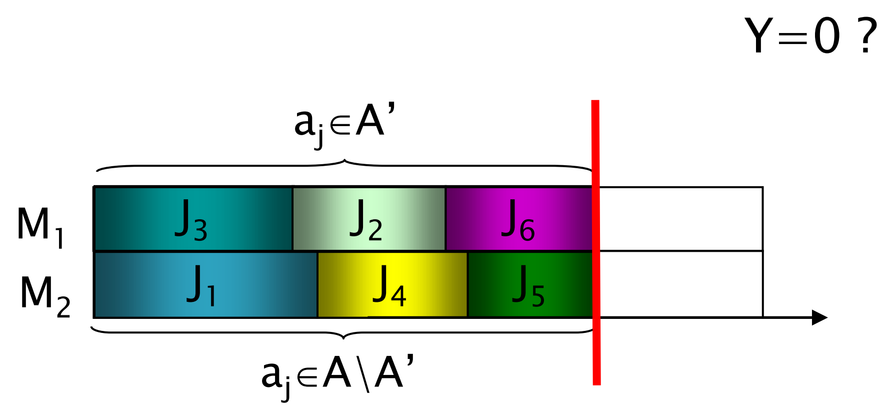


Offline Case – $P2|d_j=d|Y$

- NP–hard due to the transformation from the Partition Problem

Partition Problem $\propto$ Decision counterpart of $P2|d_j=d|Y$





$$\sum_{a_j \in A'} s(a_j) = \sum_{a_j \in A \setminus A'} s(a_j) \ ?$$

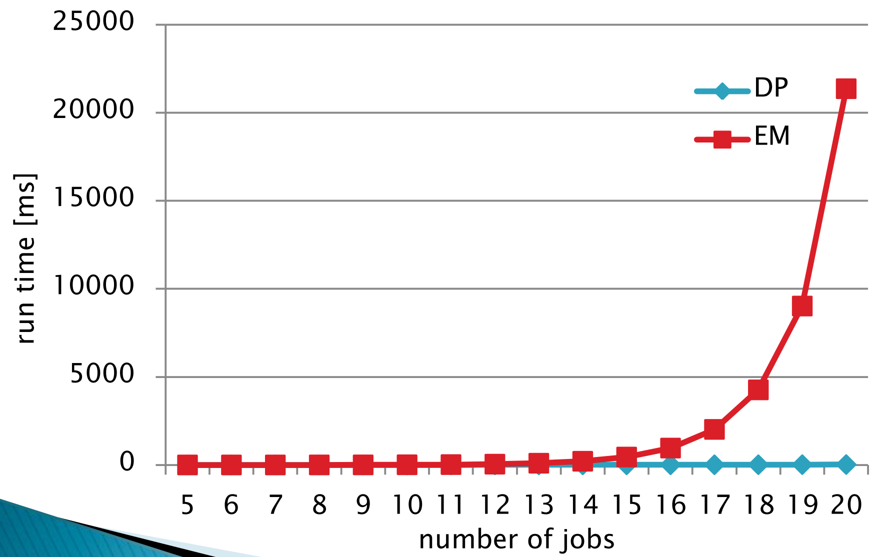
$$d = \frac{1}{2} \sum_{a_j \in A'} s(a_j)$$

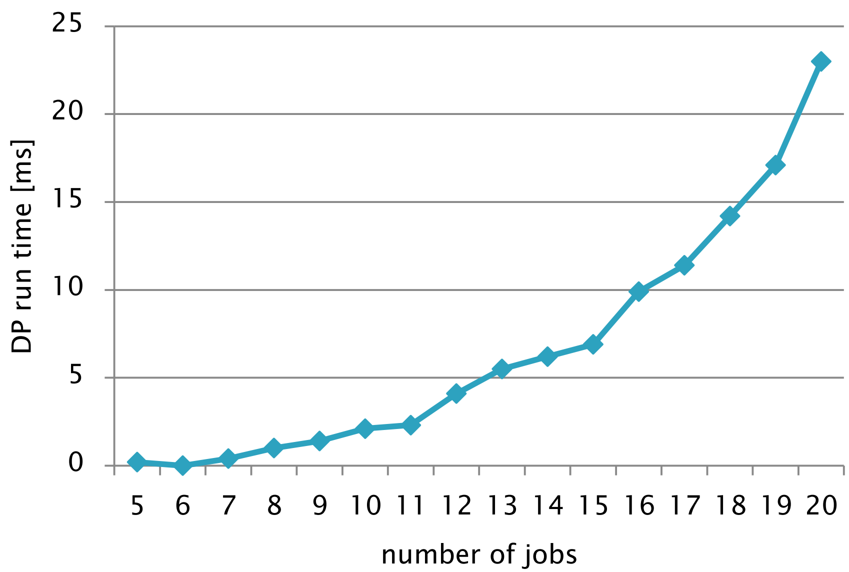
Offline Case – $P2|d_j=d|Y$

- binary NP-hard due to the existence of pseudopolynomial dynamic programming $O(nd^2)$

$$f(j, A, B) = \min\{ f(j-1, \max\{0, A-p_j\}, B) + \max\{0,p_j-A\}, \; f(j-1, A, \max\{0, B-p_j\}) + \max\{0,p_j-B\} \}$$

$$Y^* = f(n, d, d)$$





Offline Case – $P2|d_j=d|Y$

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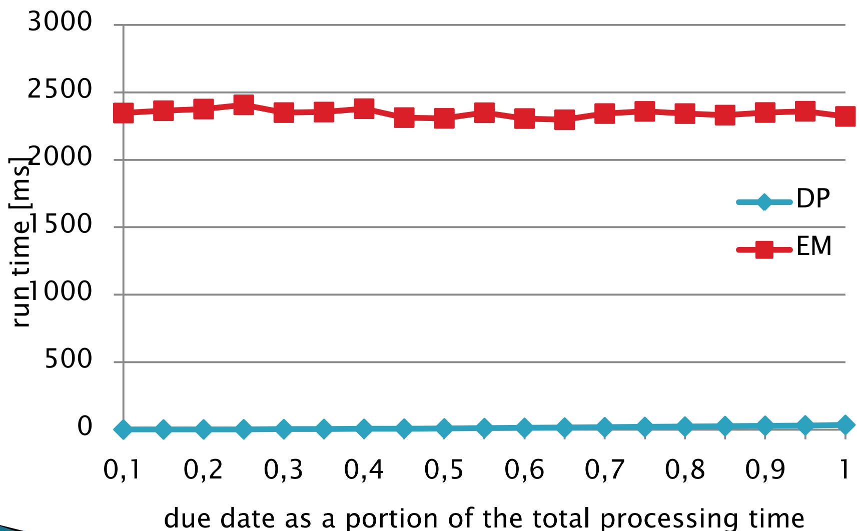
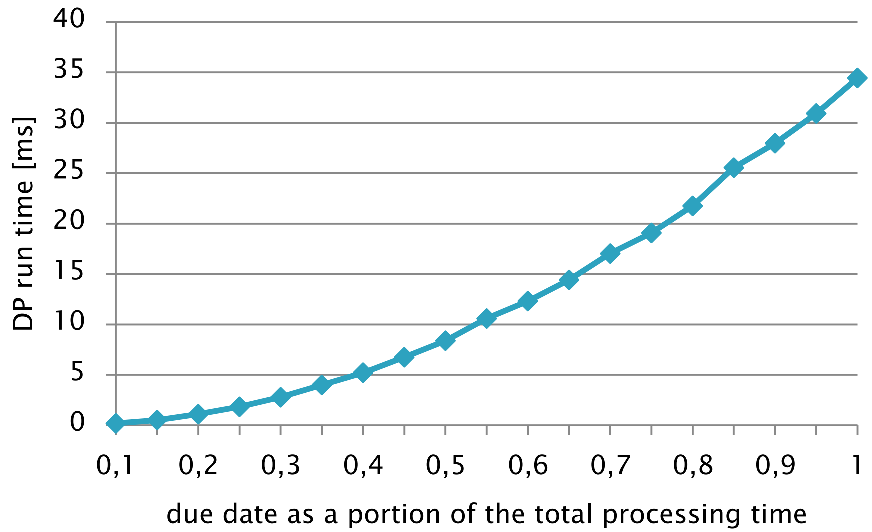
$$f(j-1, A, \max\{0, B-p_j\}) + \max\{0,p_j-B\} \}$$

$$Y^* = f(n, d, d)$$





Offline Cases – $P|d_j=d|Y$

- $P|d_j=d|Y$ is unary NP-hard
  due to the transformation from 3-Partition Problem

Online Case – $P|d_j=d$, online over list$|Y$

- to evaluate online methods the concept of early work is used
- minimizing late work – maximizing early work for a common due date is similar to bin packing problem
- online algorithm – Extended First Fit with finite competitive ratio

$$r_m = \frac{\sqrt{2m^2 - 2m + 1} - 1}{m - 1}$$

Online Case – $P|d_j=d$, online over list$|Y$

- Extended First Fit with competitive ratio r_m
  - set $t = 1$, initialize machine loads ($L^i_0=0$)
  - when job J_t with processing time p_t arrives assign it to the first fitting machine i.e. not violating assumed ratio ($L^i_{t-1}+p_t \leq r_m d$)
  - update machine loads and set $t=t+1$
- proof of competitive ratio based on case analysis

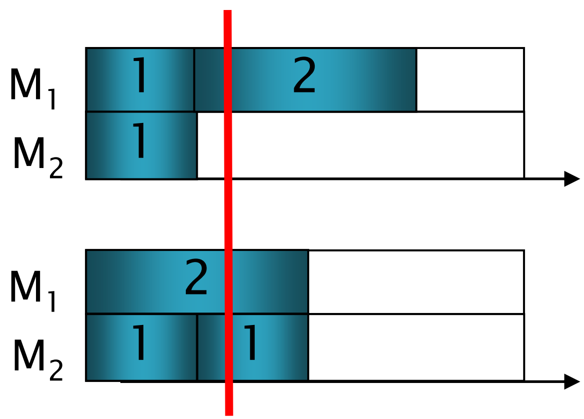
Online Case – P2|d_j=d, online over list|Y

- proof of lower bound of competitive ratio

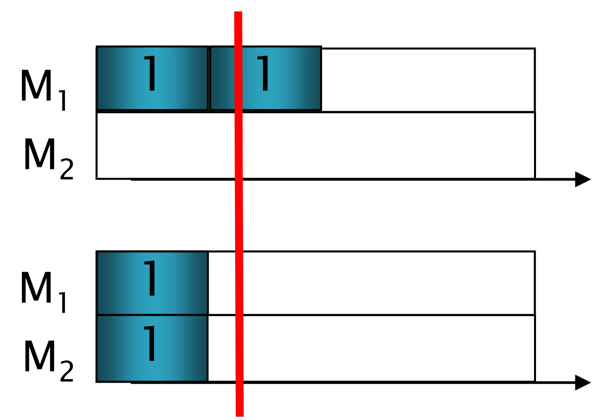
$$\sqrt{5} - 1 \approx 1.236$$

based on classical adversary sequence

$$d = \frac{\sqrt{5}+1}{2}$$



$$\frac{X^*}{X} = \frac{2d}{d+1} = \sqrt{5} - 1$$



$$\frac{X^*}{X} = \frac{2}{d} = \sqrt{5} - 1$$

Online Case – $P2|d_j=d$, online over list$|Y$

- Algorithm Extended First Fit with competitive ratio

$$r_m = \frac{\sqrt{2m^2 - 2m + 1} - 1}{m - 1}$$

is optimal for m=2
(r_2 equals to the lower bound $\sqrt{5}-1$)

List Algorithms

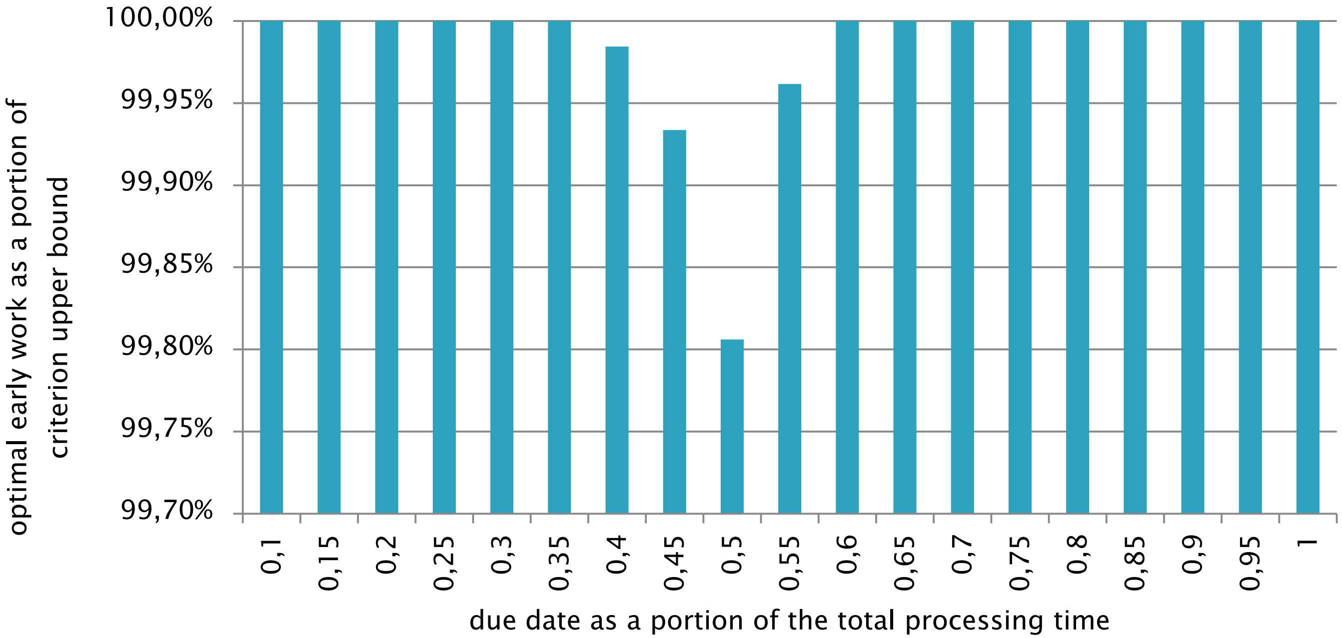
offline algorithms

- assigning jobs to the machine with the minimum load in
  - shortest processing time (SPT) order
  - longest processing time (LPT) order

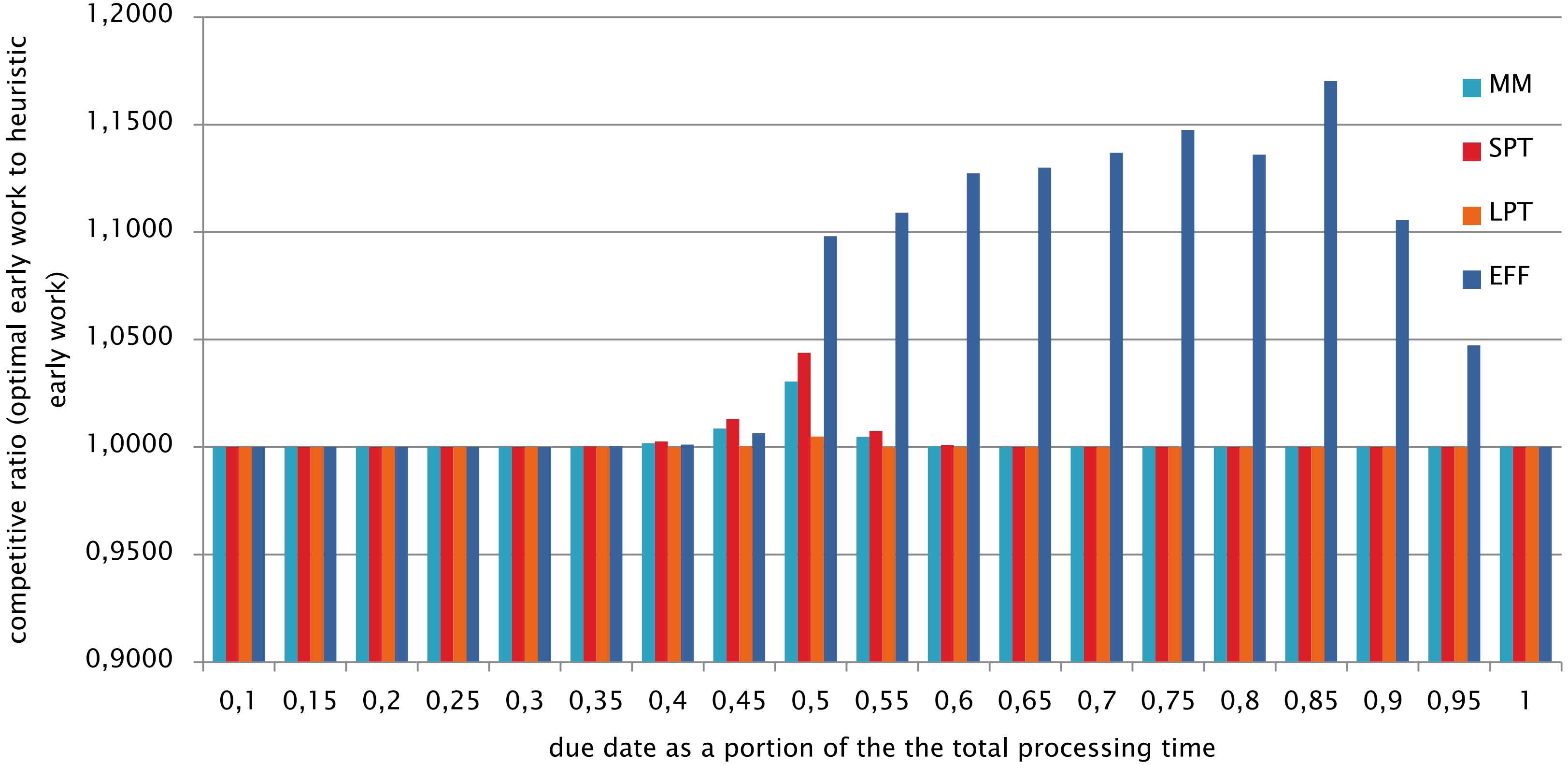
online algorithms

- assigning jobs in the input order
  - to the machine with minimum makespan (MM)
  - according to Extended First Fit (EFF)

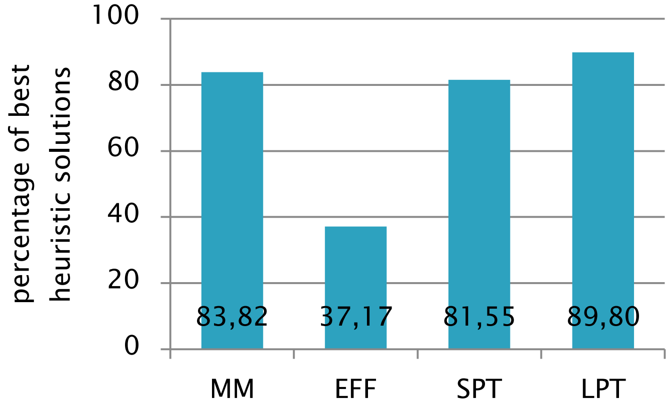
List Algorithms

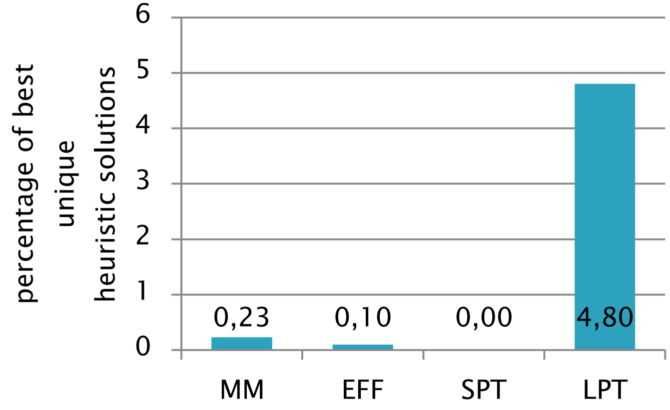


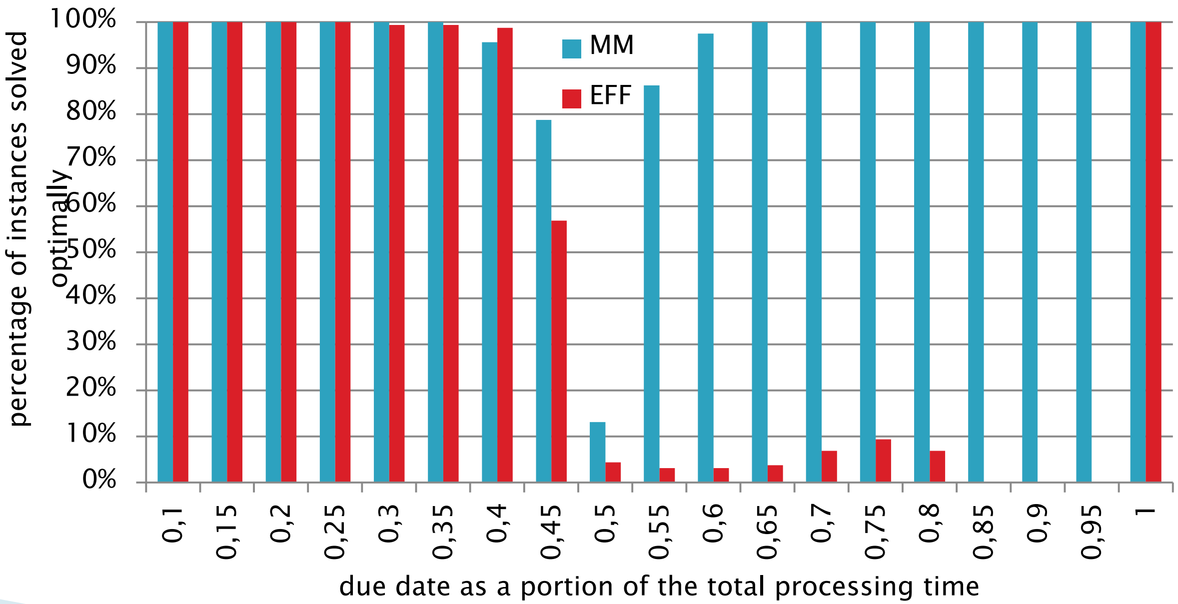
List Algorithms



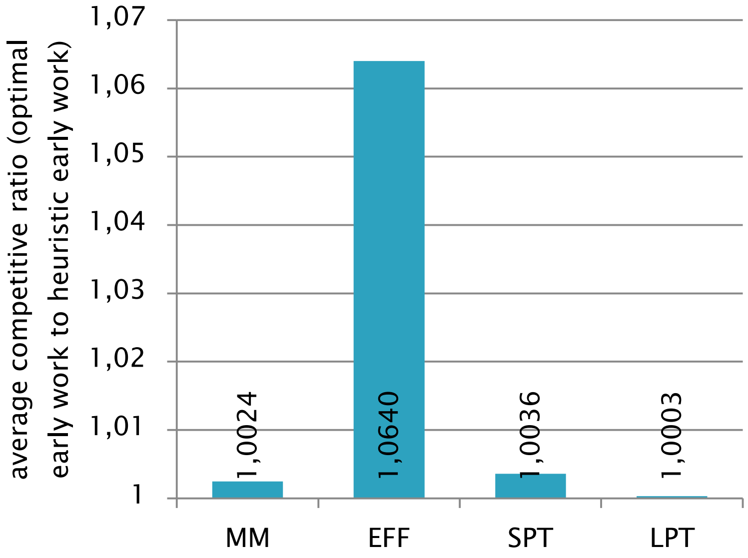
List Algorithms



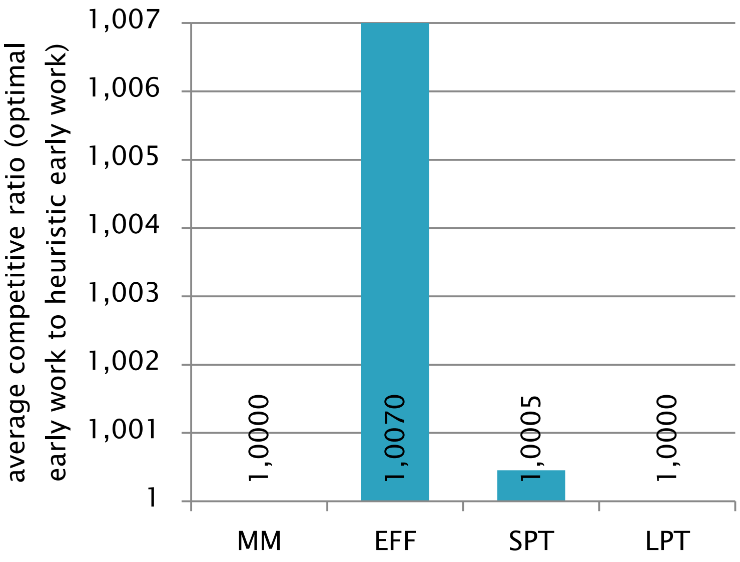




List Algorithms

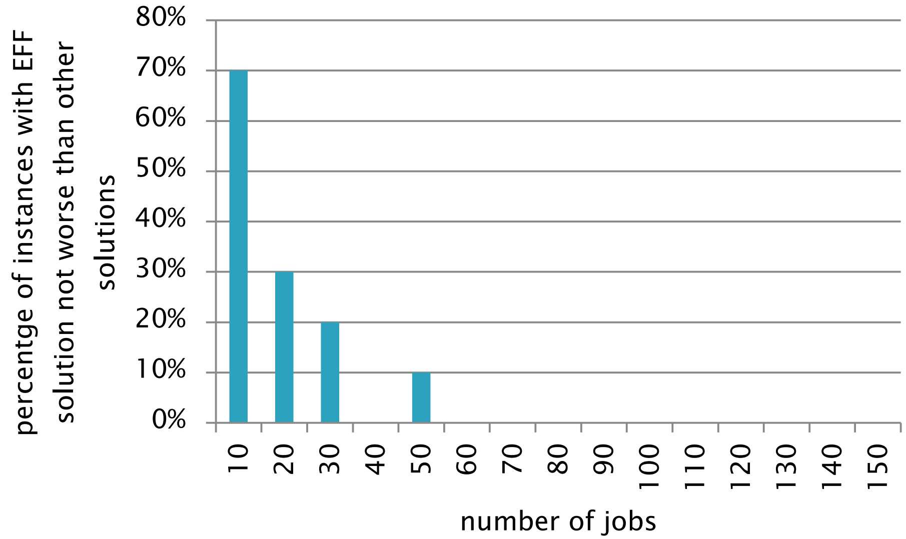


small instances n≤20



large instances n≤150

List Algorithms



Conclusions

- offline scheduling
  - $P2|d_j=d|Y$ is binary NP-hard
  - $P|d_j=d|Y$ is unary NP-hard
- online scheduling
  - online algorithm with finite competitive ratio for $P|d_j=d$, online over list$|Y$
  - optimal for $P2|d_j=d$, online over list$|Y$

- X.Chen, X.Han, M.Sterna, J. Blazewicz, Scheduling on parallel identical machines with late work criterion: Offline and online cases, Journal of Scheduling (2015), 1-8.

- simple list algorithms are very effficient
- most instances are trivial

Future research

- formulating dominance relations for two-machine case
- constructing approximation algorithms for offline case
- extending theoretical results for the problem with a given number of machines
- studying other scheduling problems with late work criterion

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Thank you for your attention!