Worst Case Bound of LRF Schedule for Fully Parallel Jobs

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Outline

- Introduction
- Formulation
- Contribution
- Research Work
  - Special case: Instance of jobs with equal density
  - General case: Instance of jobs with arbitrary weights
- Conclusion & Future Work

Introduction

1. Many companies have containers to be shipper out.
2. A cargo ship can delivery C containers per journey.
3. As long as one container can not be shipped out today, the company has to wait another day.

How to delivery the containers from different companies to minimize the total completion time?

Formulation

- Given a set J of n jobs, for each job j,
  - *Fully Parallel*, processed on any machine at any moment.
  - s_j, the workload, $s_j \in \mathbb{N}$.
  - w_j, the weight (importance), $w_j \in \mathbb{R}$.
  - released at time zero.
- Given m identical machines, for each machine i,
  - finish one job of one unit workload during one unit time.
- A feasible schedule is a table M,
  - $M(i,t)$: the job executed on machine i during time unit $[t-1,t)$,
  - Job completion time $C_j = \max_{M(i,t)=j} t$.
- Objective: minimize $T = \sum_{j \in J} w_j C_j$.

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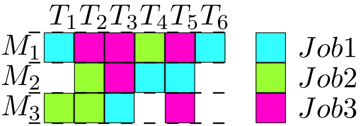

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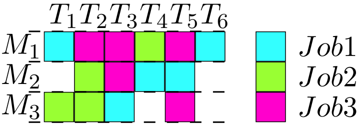

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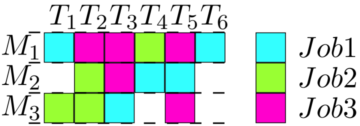

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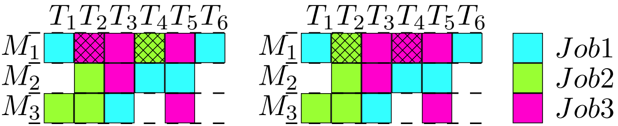

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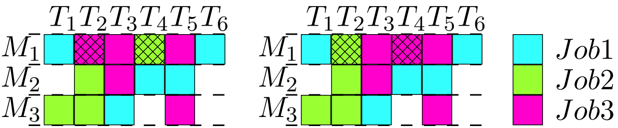

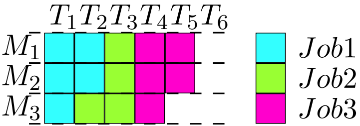

Contribution

- Related works: first introduced by Zhang et al. [TCS 2013]
  - Strongly NP-hard when m is input.
  - Proposed a 2-approximation algorithm, *Largest-Ratio-First* (LRF) algorithm.
- LRF algorithm: schedule the job with largest ratio w/s first.
- Our contribution: the LRF algorithm is α - approximation,
  - $\alpha = 1 + \frac{i+(i-2n/m)}{i(i+1)}$ for instance of jobs with equal density $w_j/s_j = 1$, where $i = \lceil \frac{2n}{m} \rceil$.
    - i) $1 + \frac{1}{i+1} \leq \alpha < 1 + \frac{1}{i}$.
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Instance of Jobs With Equal Density (overview)

- Equal density: $w_j = s_j, \ \forall j \in J$.
  - LRF algorithm = arbitrary order of jobs.
  - $\alpha(J) = \frac{\max_{S \in permutation(J)} T(S,J)}{\min_{S \in permutation(J)} T(S,J)}$
- Goal: for fixed value of n and m, find maximum α & corresponding instance J^*.

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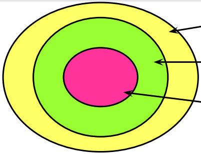

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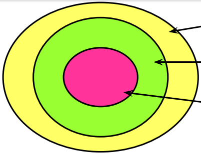

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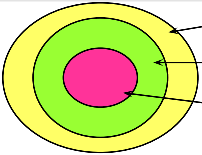

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Splitting of Jobs

Definition

Free job: be executed within one unit time;
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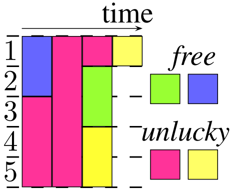

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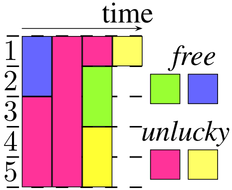

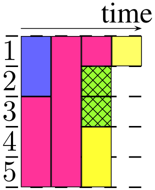

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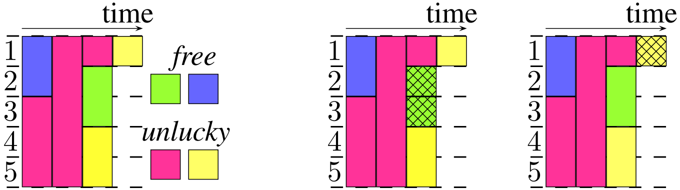

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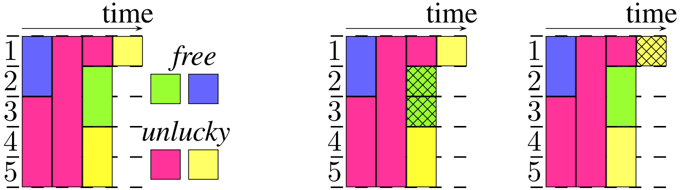

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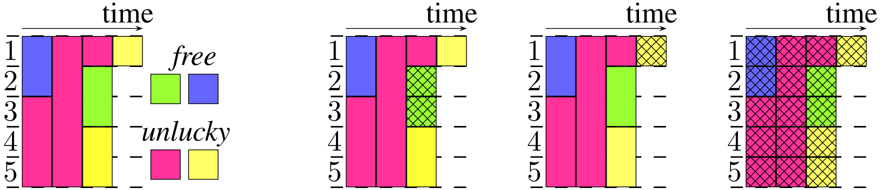

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For any feasible schedule S, $T(S, J) \geq T(J^{unit})$.

Properties of Organized Instance

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An *organized instance* $J(y, z, k) = m^y + k^1 + 1^z$ s.t. $L(J) > m(y+1)$, where $n = y + z + 1$, $1 < k \leq m$, $0 < z < n$ $(y, z, k \in \mathbb{N})$.

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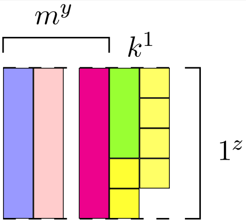

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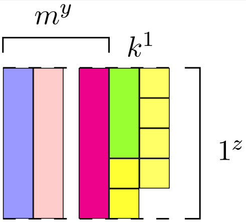

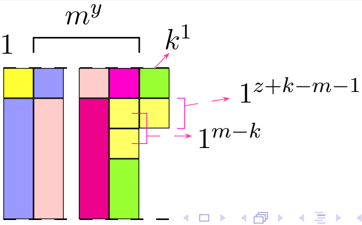

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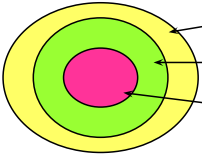
Instance Achieving Maximum Approximation Ratio

Next, we prove that

$$\alpha(\mathfrak{J}_{org}^{(m,n)}) = \alpha(\mathfrak{J}_{one}^{(m,n)}) = \alpha(\mathfrak{J}^{(m,n)})$$



$$\mathfrak{J}^{(m,n)} = \{J \mid \forall j \in J \;\; w_j = s_j,\ |J| = n\}$$

$$\mathfrak{J}_{one}^{(m,n)} = \{J \mid \forall j \in J \;\; 1 \le s_j \le m,\ J \in \mathfrak{J}^{(m,n)}\}$$

$\mathfrak{J}_{org}^{(m,n)}$ contains all the *organized instances* of n jobs

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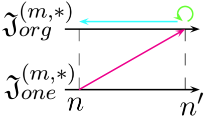

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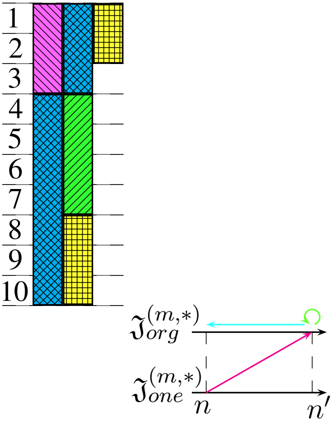

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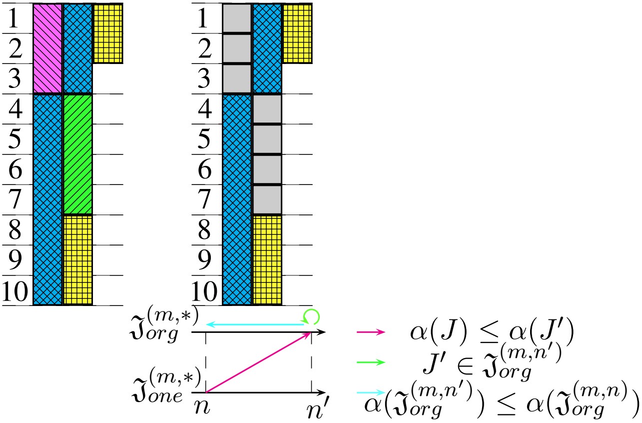

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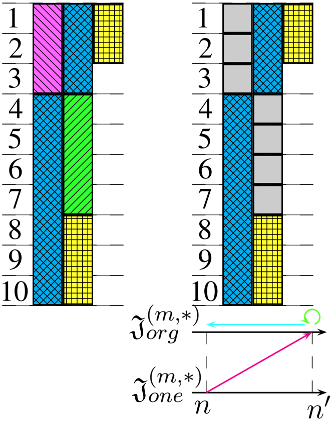

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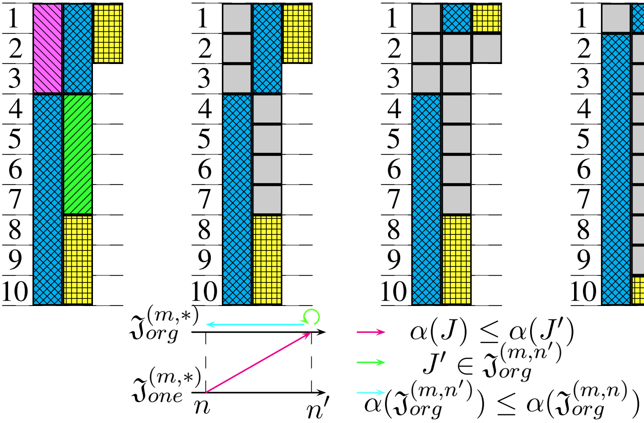

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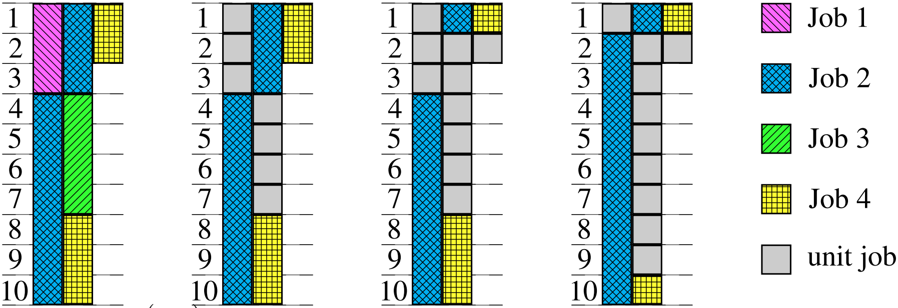

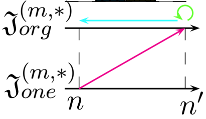

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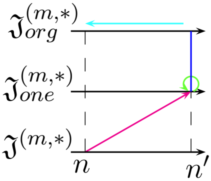

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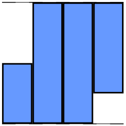

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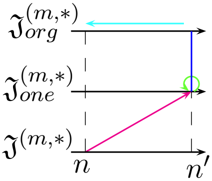

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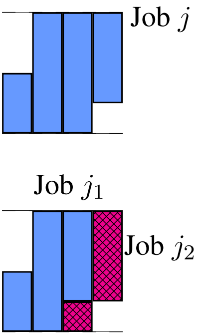

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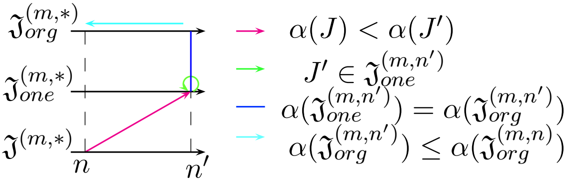

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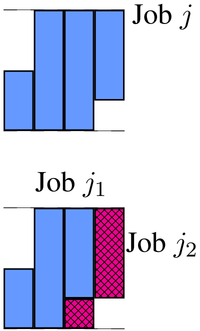

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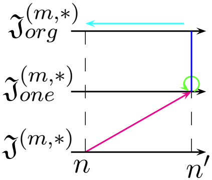

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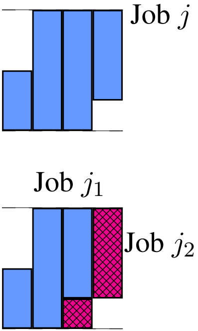

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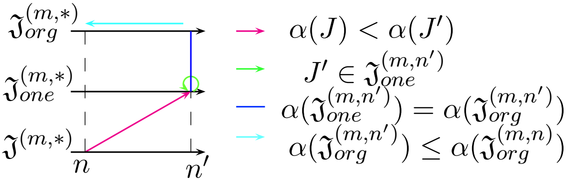

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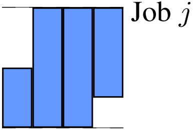

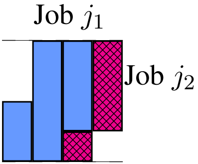

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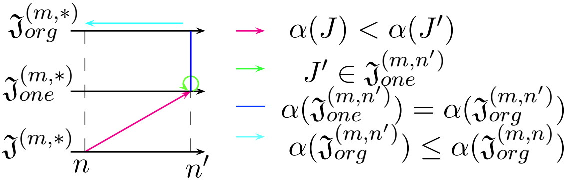

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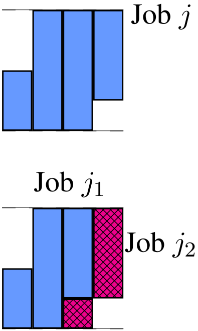

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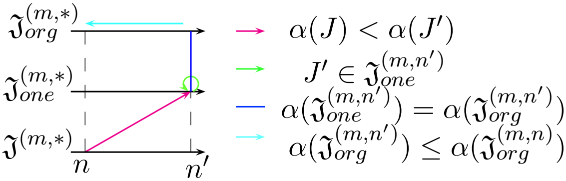

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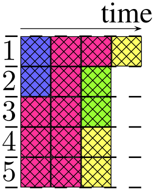

Approximation Ratio Bound (Organized Instance)

- Define function $g(a,b) = \frac{am+b-n}{ma(a+1)/2+b(a+1)}$ $(a > 0,\ b \geq 0)$.

- Define $i = \lceil \frac{2n}{m} \rceil$,

$$g(a,b) \leq g(i,0),\ \forall a > 0, 0 \leq b < m, a, b \in \mathbb{N}$$

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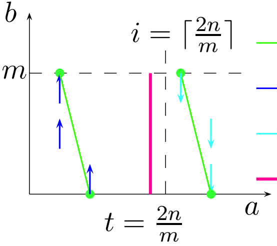

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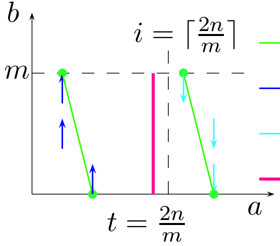

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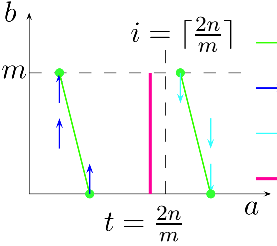

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Lemma

For any organized instance $J \in \mathfrak{J}_{org}^{(m,n)}$, $L(J) = im$ *if and only if* $i \leq n + 1 - m$, *where* $i = \lceil \frac{2n}{m} \rceil$.

- how about $i > n + 1 - m$, $L(J) = im - 1,\ im + 1, ..?$
- Group the instance, in each group (region) of instance we find the tight bound.
- Roughly, for region B_i, $B_i = \{(m, n) \mid i = \lceil \frac{2n}{m} \rceil, m \geq 3\}$

Approximation Ratio Bound (Organized Instance)

<table>
<tr><th>Regions</th><th>Approximation Ratio</th><th>function $g(a,b)$</th></tr>
<tr><td>$B_0 = \{m = 2\}$</td><td>$1 + \frac{n-1}{n^2}$ (tight)</td><td>$g(n-1, m-1)$</td></tr>
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<tr><td>$B_2^* = \{m = 2n-1, m \geq 3\}$</td><td>$1 + \frac{m-1}{2m-1}$ (tight)</td><td>$g(i-1, n-1)$</td></tr>
<tr><td>$B_2 = \{n \leq m \leq 2n-2, m \geq 3\}$</td><td>$1 + \frac{2m-n+1}{3m+3}$ (tight)</td><td>$g(i, 1)$</td></tr>
<tr><td>$B_3^* = \{m = n-1, m \geq 3, n \neq 4\}$</td><td>$1 + \frac{2m-2}{6m-3}$ (tight)</td><td>$g(i-1, m-1)$</td></tr>
<tr><td>$B_3 = \{\frac{2n}{3} \leq m \leq n-2, m \geq 3\}$</td><td rowspan="3">$1 + \frac{2(im-n)}{i(i+1)m}$ (tight)</td><td rowspan="3">$g(i, 0)$</td></tr>
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Table: Approximation ratio bound of instances in different regions.

Approximation Ratio Bound (Organized Instance)

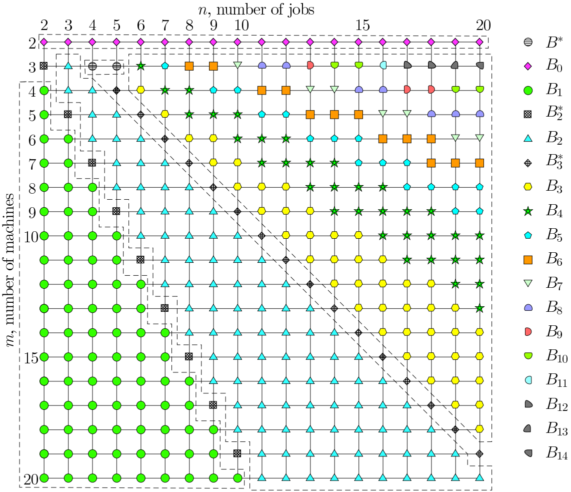

Figure: Example of the region division for $2 \le n, m \le 20$

Instance of Jobs with Arbitrary Weights

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$\forall J$, let $J^{(e)} = \{(w'_j, s_j) \; | w'_j = s_j, j \in J\}$ be the corresponding job set of J.

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Theorem

$\forall J,\ \alpha(J) \leq \alpha(\mathfrak{J}^{(m,2)}) = 1 + \frac{m-1}{m+2}$.

Conclusion

- We prove that LRF algorithm is α - approximation ($\alpha < 2$).
- For instance of jobs with equal density $w_j/s_j = 1$,
  - $\alpha = 1 + \frac{i+(i-2n/m)}{i(i+1)}$, where $i = \lceil \frac{2n}{m} \rceil$
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