Flow shop scheduling with Delivery coordination

Presented by
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Contents

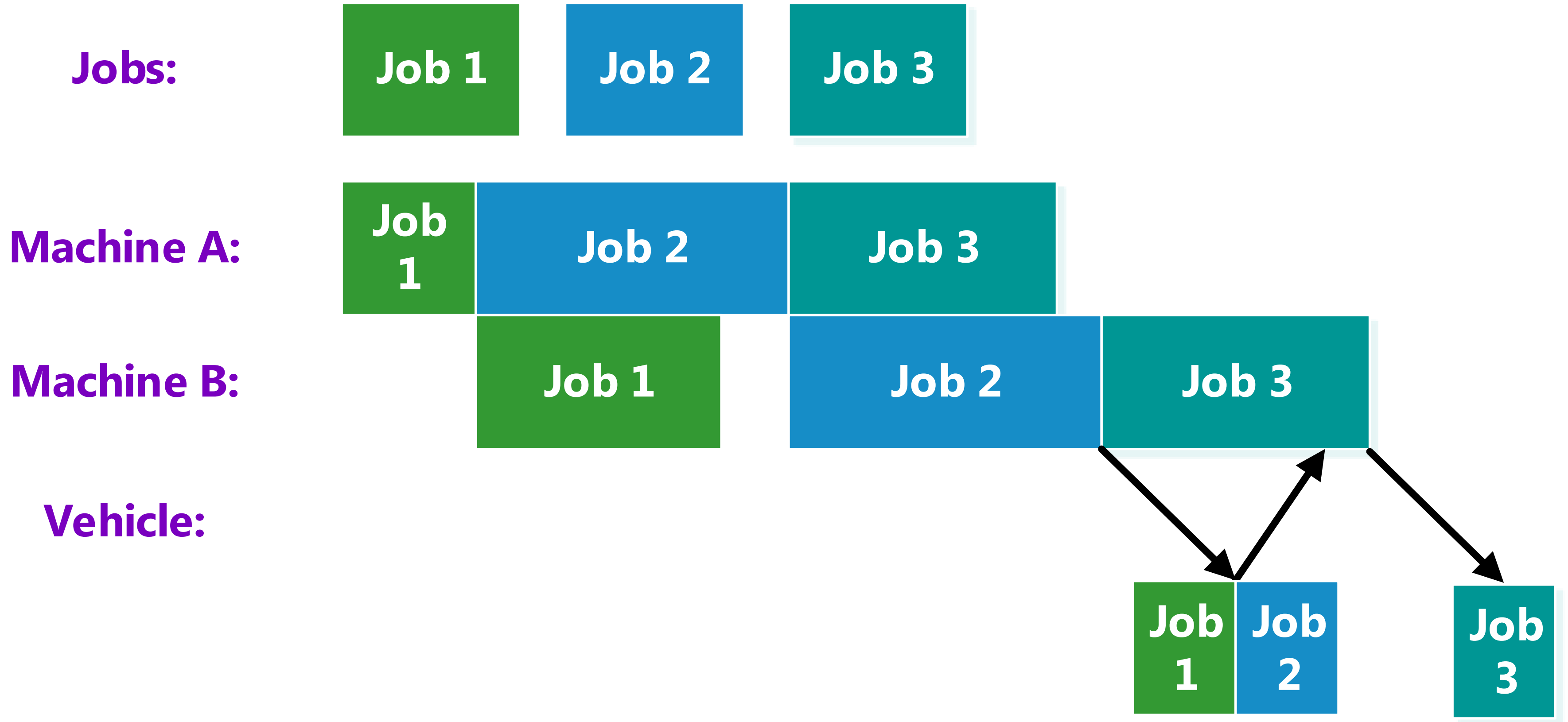
The Model

$$F2 \rightarrow D|v = 1, c|C_{\max}$$

$F2$	**Flowshop scheduling with two machines**
D	**Destination**
$v = 1$	**One transporter**
$c \geq 1$	**Capacity $\geq$ 1**
C_{max}	**the object of minimize the makespan**

An Example

$$F2 \rightarrow D|v = 1, c = 2|C_{\max}$$



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Main Results

1	Prove $\mathrm{F2} \rightarrow \mathrm{D}|\mathrm{v}=1, \mathrm{c}=2|\mathrm{C}_{\max}$ is strongly NP-hard
2	Study three special cases of the model and give related polynomial optimal algorithms
3	Give PTAS for $\mathrm{F2} \rightarrow \mathrm{D}|\mathrm{v}=1, \mathrm{c}|C_{\max}$

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Previous results

Author	Year	Problem	Complexity
Lee & Chen[1]	2001	$F2 \rightarrow D\|v=1,c=1\|C_{\max}$	SNP
		$F2 \rightarrow D\|v=1,c\geq 4\|C_{\max}$	SNP
Yuan et.al.[2]	2007	$F2 \rightarrow D\|v=1,c=2\|C_{\max}$	Binary NP-hard
		$F2 \rightarrow D\|v=1,c=3\|C_{\max}$	SNP

[1] Chung-Yee L, Chen Z. Machine scheduling with transportation considerations[J]. Journal of Scheduling, 2001, 4(1):3-24.
[2] Yuan J, Soukhal A, Chen Y, et al. A note on the complexity of flow shop scheduling with transportation constraints[J]. European Journal of Operational Research, 2007, 178(3):918-925.

Our first results

$$F2 \rightarrow D|v = 1, c = 2|C_{\max}$$

Just binary NP-hard ?

NP-hard in strong sense?

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Main Idea

4-partition problem

equivalent

$$F2 \rightarrow D|v = 1, c = 2|C_{\max}$$

4-Partition problem

Inpute: $4m$ positive integers $a_1, a_2, \dots, a_{4m}$ with $\sum_{i=1}^{4m} ai = mb$
$and\ b/5 < a_i < b/3$, for $i = 1, \dots, 4m.$

Output: Find the partition $H_1, H_2, \dots, H_m$ such that
$|H_j| = 4$ and $\sum_{i \in H_j} a_i = b$ for $j = 1,2, \dots, m.$

The above 4-PARTITION problem is a Strong NP-hard problem [3].

[3] Garey M R, Johnson D S. Computers and Intractability: A Guide to the Theory of NP-Completeness[C]// W.H. Freeman and Company. 1979.

Instance 4-partirion to $F2 \to D$

First, use the 4-partirion instance to construct a jobs instance of $F2 \to D$.

Big	Job 0:	$A(0) = 0,$	$B(0) = 2.1b$
Small	Jobs i:	$A(i) = 0.1b + 0.8a_i,$	$B(i) = 0.9a_i,$ for $i = 1, \ldots, 4m$
Big	Jobs j:	$A(j) = 1.6b,$	$B(j) = 2.1b,$ for $j = 4m + 1, \ldots, 5m$
Tiny	Jobs k:	$A(k) = 0.2b,$	$B(k) = 0,$ for $k = 5m + 1, \ldots, 6m + 1$

The delivery time is b

4-partirion to $F2 \to D$

Lemma 1: Assume $H_1, H_2, \ldots, H_m$ is a solution for 4-Partition, then there is a feasible schedule S with makespan no bigger than $3mb + 3.1b$

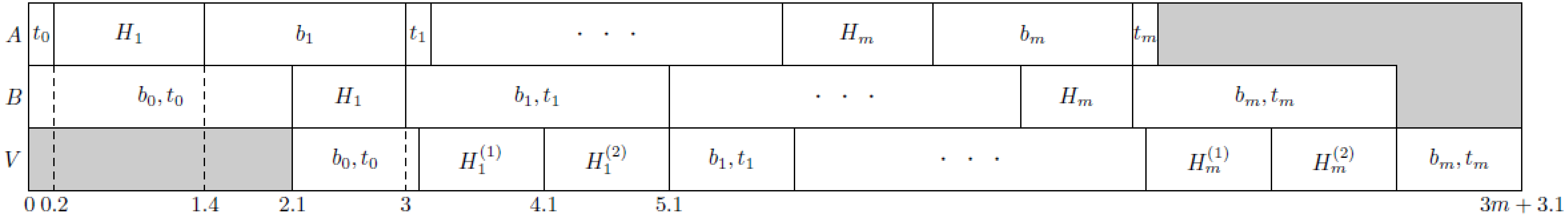

Fig. 1. A feasible schedule S with $C_{\max}(S) = (3m + 3.1)b$

4-partirion to $F2 \to D$

Lemma 2. *If $C_{\max}(S) \leq y = 3mb + 3.1b$, then there is no idle time on machine B and the first job that processed on A and B is big job 0.*

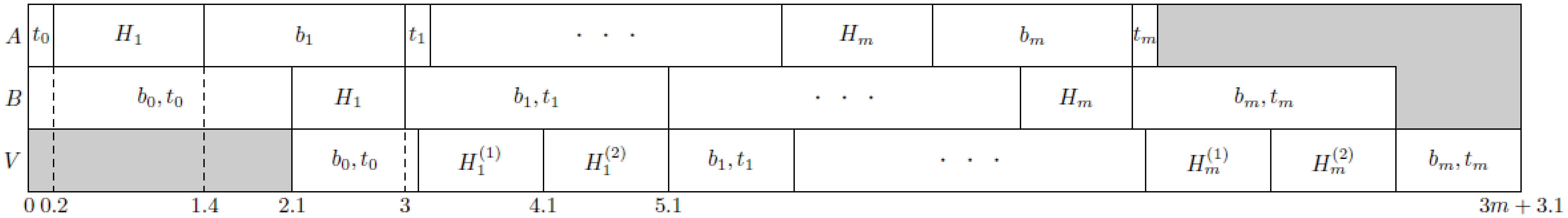

Fig. 1. A feasible schedule S with $C_{\max}(S)=(3m + 3.1)b$

4-partirion to $F2 \rightarrow D$

Lemma 3. *The transporter V is idle during time span $[0, 2.1b]$ and other time is always busy. And it carries two jobs each time. The second job processed on B is a tiny job.*

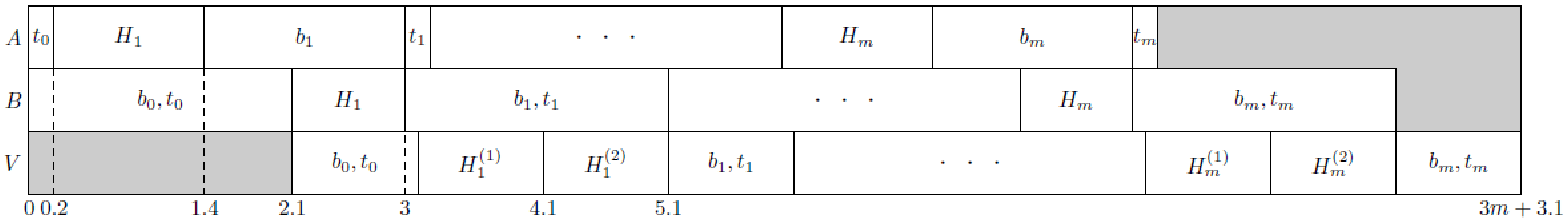

Fig. 1. A feasible schedule S with $C_{\max}(S) = (3m + 3.1)b$

$F2 \to D$ to 4-partirion

Lemma 4. *We call big job 0 and a tiny job on B the first batch. If* $C_{max}(S) \leq y$, *then machine B must complete four jobs between the first batch and the second big job.*

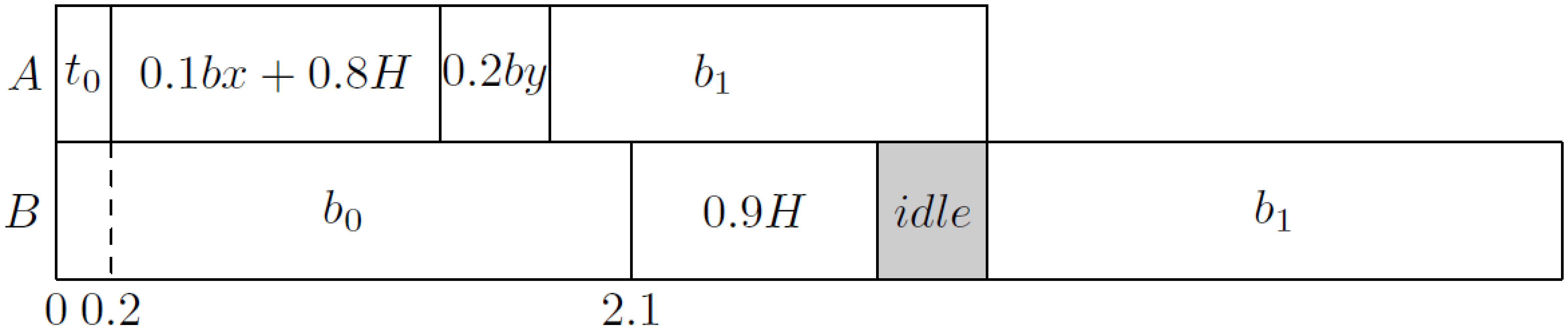

Fig. 2. A schedule when $x + y \geq 5$

$F2 \rightarrow D$ to 4-partirion

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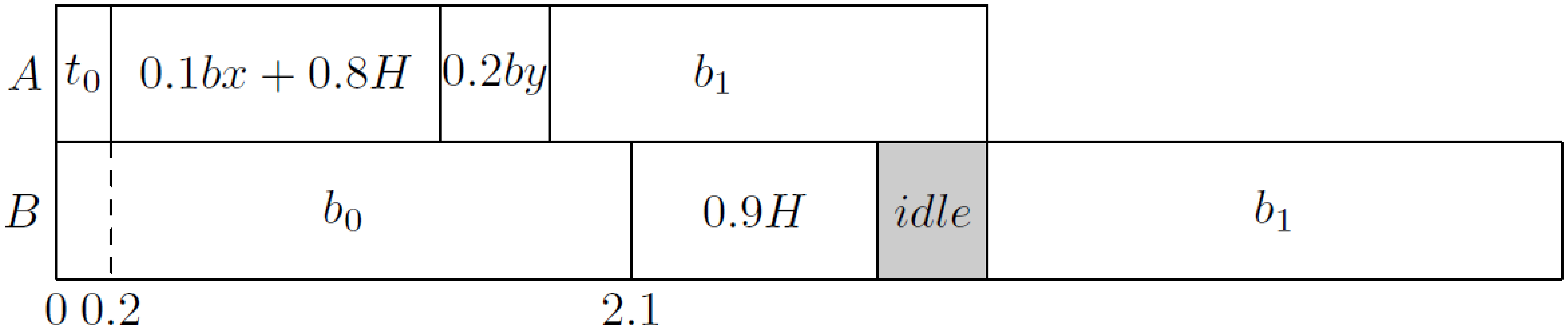

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$F2 \rightarrow D$ to 4-partirion

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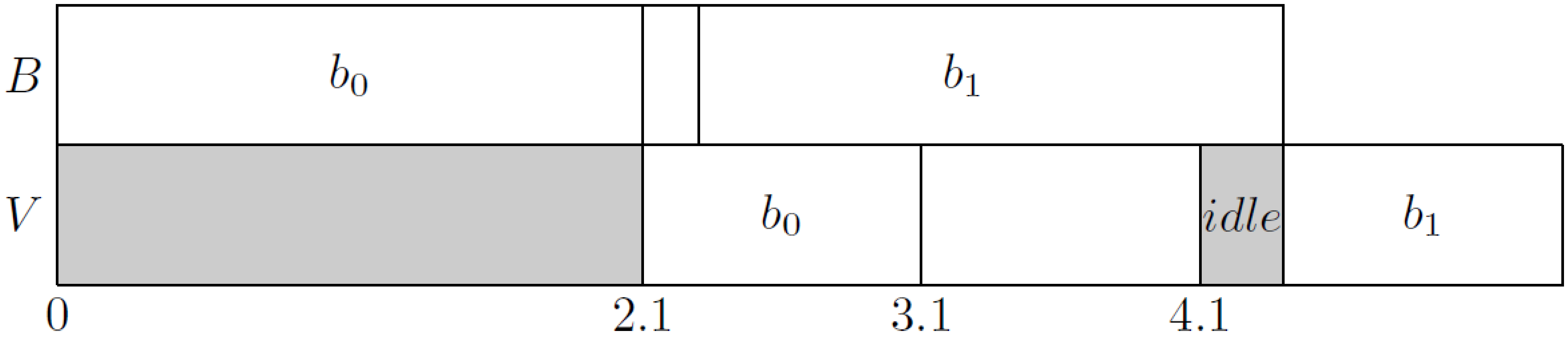

Fig. 3. A schedule when $x + y = 2$

$F2 \rightarrow D$ to 4-partirion

Lemma 5. *The four jobs between the first batch and big job b_1 on B are all small jobs. Let H be the set of elements in 4-PARTITION which correspond with the four small jobs. Then the total size in H is exactly b.*

Lemma 6. *If $C_S \leq 3mb + 3.1b$, then the instance of 4-PARTITION has a feasible solution.*

Our first results

By lemma 1 and lemma 6, we can find that

$4 - partition\ problem \leftrightarrow F2 \rightarrow D|v = 1, c = 2|C_{\max}$

Then we can get the following theorem:

Theorem 1. $F2 \rightarrow D|v = 1, c = 2|C_{\max}$ *is a NP-hard problem in strong sense.*

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Our Second results

Consider three special cases of

$$F2 \rightarrow D|v = 1, c \geq 1|C_{\max}$$

and give polynomial optimal Algorithms.

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Three special cases

Problem	Time-Complexity
$F2 \to D \mid v=1, c \geq 1, chain \mid C_{\max}$	$O(1)$
$F2 \to D \mid v=1, c \geq 1, B(i)=p \mid C_{\max}$	$O(nlogn)$
$F_2 \to D \mid v=1, c \geq 1, \max_{1 \leq i \leq n} \{A(i)\} \leq \min_{1 \leq i \leq n} \{B(i)\} \mid C_{\max}$	$O(n^2)$

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The Relations

$$F2 \rightarrow D|v = 1, c \geq 1, chain|C_{max}$$

call

$$F2 \rightarrow D|v = 1, c \geq 1, B(i) = p|C_{max}$$

call

$$F_2 \rightarrow D \,|\, v = 1, c \geq 1, \max_{1 \leq i \leq n}\{A(i)\} \leq \min_{1 \leq i \leq n}\{B(i)\} \,|\, C_{\max}$$

$F2 \to D|v=1, c \ge 1, chain|C_{max}$

The jobs' processing order on machines A,B has been given before the scheduling.

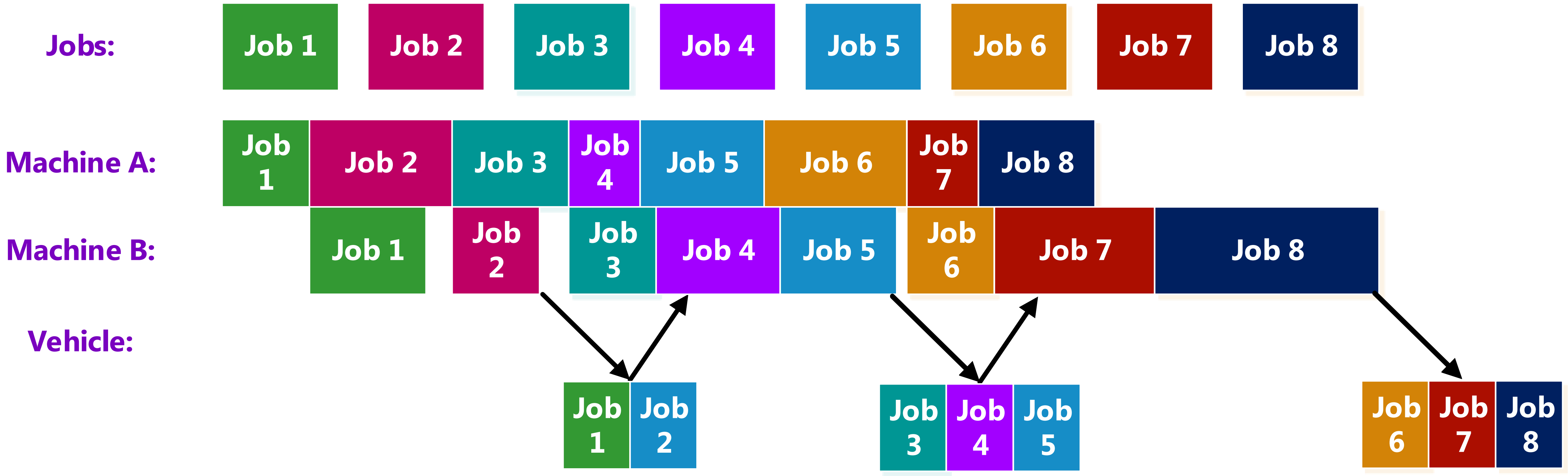

Fig 2

Conclusion

(1) prove $F2 \rightarrow D|v = 1, c = 2|C_{\max}$ is strongly NP-hard

(2) study three special cases of the model and give related polynomial optimal algorithms

(3) Give PTAS for $\mathrm{F2} \rightarrow \mathrm{D}|\mathrm{v} = 1, \mathrm{c}|C_{\max}$

Q&A

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